# Advanced Topics in Stochastic Analysis
# - Introduction to Schramm-Loewner evolution

---

**Mondays 12–14** and **Thursdays 8–10** in *Endenicher Allee 60 - SemR 1.008*

---

Schramm-Loewner Evolution (SLE) is a key concept when studying the geometry of random structures. It lies at the interface between probability, geometry, and analysis, combining beautiful theory of conformal mappings to stochastic analysis and properties of Brownian motion. The SLE is a random fractal curve in the plane, whose most famous applications include the understanding of geometric properties of statistical models in two dimensions (random walks, percolation, Ising model, Gaussian free field, ...).

The goal of the course is to provide an introduction to the definition, properties, and applications of the SLE.

We will cover some background material on complex analysis and stochastic analysis, when needed (the precise plan will depend on the participants' background knowledge and wishes). Along the way, we develop Loewner's theory for growth processes encoded in conformal maps. Then we define the SLE processes and prove their basic properties. Time permitting, we discuss further properties and applications of the SLE.

**Rough Plan for the Lectures:**

- Overview and motivation
- Recap of conformal maps, their boundary behavior, harmonicity, and Riemann mapping theorem
- Recap of Brownian motion and basic concepts from stochastic analysis
- Growth processes encoded in conformal maps and the Loewner differential equation
- Stochastic Loewner flows and definition of Schramm–Loewner evolution ($\mathrm{SLE}_\kappa$)
- Conformal invariance and domain Markov property, Schramm's theorem
- The $\mathrm{SLE}_\kappa$ as a random non-self-crossing continuous fractal curve
- Hitting probabilities, phase transitions in $\kappa \geq 0$, locality, and conformal restriction property
- Examples of SLEs in statistical physics and probability theory

**Prerequisites:**

Probability theory and some familiarity with basic complex analysis. Stochastic analysis will be needed towards the end. This SLE course can also be taken in parallel with "Foundations in Stochastic Analysis".

**Literature:**

- Nathanaël Berestycki and James Norris. Lecture notes on SLE.
  `http://www.statslab.cam.ac.uk/~james/Lectures/sle.pdf`
- Wendelin Werner. Random planar curves and Schramm-Loewner evolutions.
  Notes from lectures delivered at the Saint-Flour summer school (July 2002).
  `http://arxiv.org/pdf/math/0303354.pdf`
- Antti Kemppainen. Schramm-Loewner evolution. SpringerBriefs in Mathematical Physics, 2017.
  `http://wiki.helsinki.fi/display/mathphys/sle-book`
- Gregory Lawler. Conformally Invariant Processes in the Plane. American Mathematical Society, 2005.
  `http://pi.math.cornell.edu/~lawler/book.ps`