Anomalous shock fluctuations in the asymmetric exclusion process

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Outline

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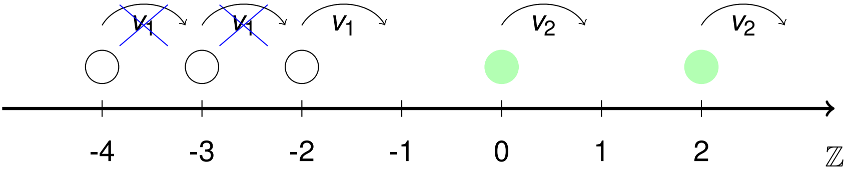
Totally asymmetric simple exclusion process (TASEP)



- **Dynamics**: particles on $\mathbb{Z}$ perform independent jumps to the right subject to the exclusion constraint
- We will also consider particle-dependent jump rates
- continous-time Markov process with state space $\{0, 1\}^{\mathbb{Z}}$

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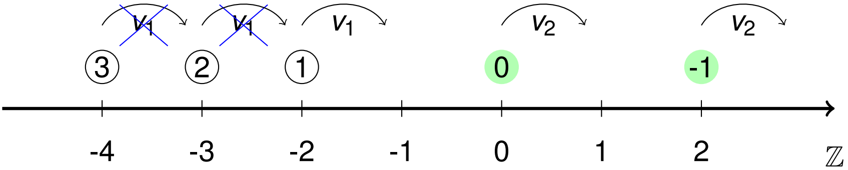

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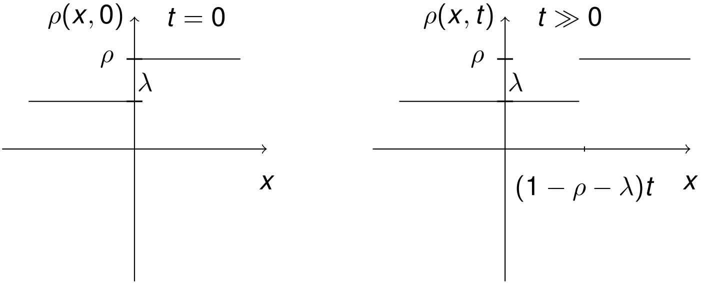
We can number particles from right to left

$$\ldots < x_3(0) < x_2(0) < x_1(0) < 0 \leq x_0(0) < x_{-1}(0) < \ldots$$

$$\ldots < x_3(t) < x_2(t) < x_1(t) < x_0(t) < x_{-1}(t) < \ldots$$

Shocks

- Discontinuities of the particle density are called **shocks**

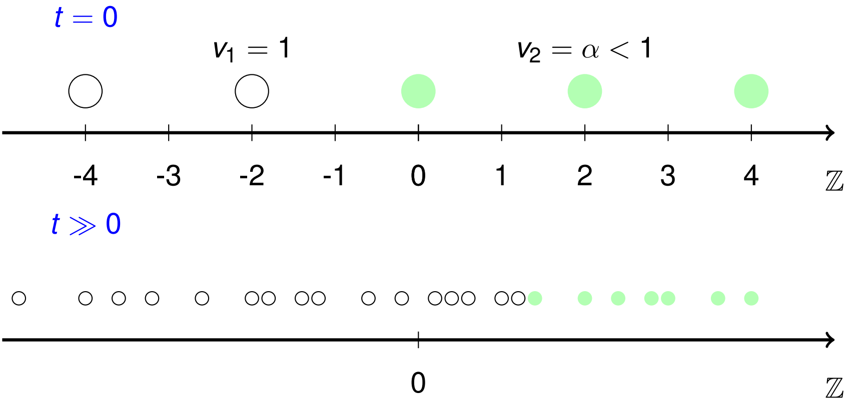


- Initial condition: $\mathrm{Ber}(\rho)$ on $\mathbb{N}$ and $\mathrm{Ber}(\lambda)$ on $\mathbb{Z}_-$.
- one can identify the shock with the position Z_t of a second-class particle initially at 0 :

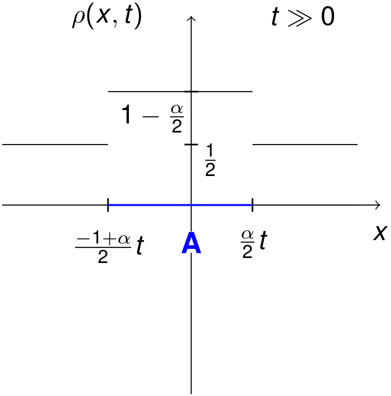
$$\lim_{t\to\infty} \frac{Z_t - vt}{t^{1/2}} \to \mathcal{N}(0,1),\ v = 1-\lambda-\rho \ \text{[see Lig'99]}$$

Question: What are the shock fluctuations for **non-random initial configuration (IC)**?

Two Speed TASEP with periodic IC



Heuristics from macroscopic continuity equation



- The last slow particle is macroscopically at position $(1-\rho)\alpha t = \frac{\alpha}{2}t$.
- Behind it is a jam region A of increased density $\rho = 1 - \alpha/2$.
- The particle ηt, with $\eta = \frac{2-\alpha}{4}$ is at the macro shock position.

Inside the constant density regions, $\eta' \neq \eta$, the fluctuations of $x_{\eta' t}$ are governed by the F_1 GOE Tracy-Widom distribution from random matrix theory and live in the $t^{1/3}$ scale.

Goal: Determine the large time fluctuations of the (rescaled) particle position $x_{n(t)}$ around the shock:

$$\lim_{t\to\infty} \mathbb{P}\left(\frac{x_{n(t)} - vt}{t^{1/3}} \leq s\right) = ?$$

where $vt = \frac{-1+\alpha}{2} t$ is the macroscopic position of $x_{n(t)}$.

For arbitrary fixed IC, the law of $x_{n(t)}$ is given as a Fredholm determinant of a kernel K_t [Borodin-Ferrari'08],

$$\lim_{t\to\infty} \mathbb{P}\left(\frac{x_{n(t)} - vt}{t^{1/3}} \leq s\right) = \lim_{t\to\infty} \det(1 - \chi_s K_t \chi_s), \quad (1)$$

The series expansion of $\det(1 - \chi_s K_t \chi_s)$ is

$$\det(1-\chi_s K_t \chi_s) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_s^{\infty} \mathrm{d}s_n \cdots \int_s^{\infty} \mathrm{d}s_1 \det\left(K_t(s_i, s_j)_{1\leq i,j\leq n}\right)$$

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We will actually translate TASEP into a different and more generic model, and determine the limit there.

Outline

Microscopic shock analysis

- We choose $\alpha = 1 - \frac{a}{2^{4/3}t^{1/3}}$, $a > 0$. For this α, $\lim_{t\to} K_t$ (modulo some prefactors) exists and is denoted by K_a
- K_a is explicitly given in terms of the Airy-function Ai
- As $a \to +\infty$, K_a is again diverging.

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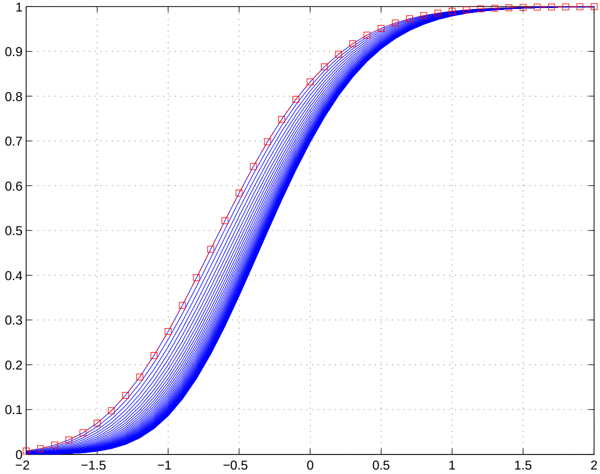
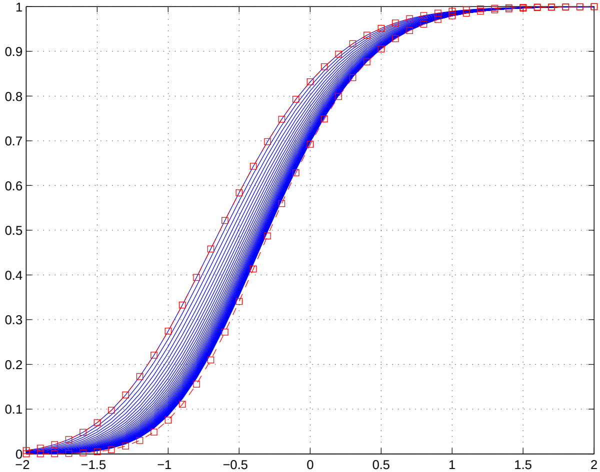
  for a getting larger so as to recover the macroscopic shock distribution
  Numerical limitations:
  $G_3(s) = NaN$ and $G_{2.1}(-3) = -5.25 \times 10^{25}$.



The red line is $F_1(2s)$ for $s = -2, -1.9, \ldots, 2$
The blue lines are $G_a(s)$ for $a = 0, 0.05, 0.1, \ldots, 2.05$



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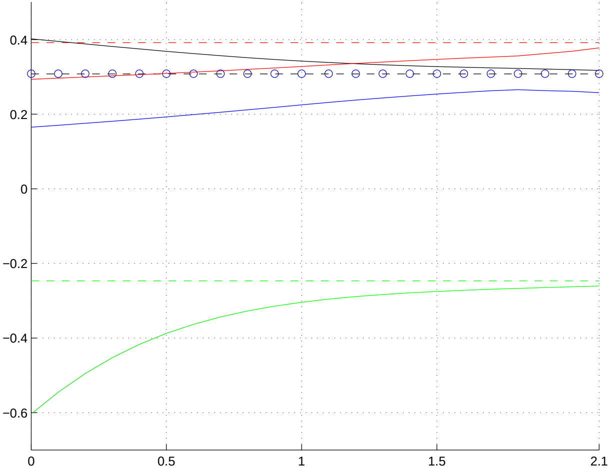
The blue lines are $G_a(s)$ for $a = 0, 0.05, 0.1, \ldots, 2.05$
For $a = 2.05$ the fit with $F_1(2s)^2$ is very good

Basic statistics of G_a



Expectation, Variance, Skewness, Kurtosis of G_a (dashed) and $F_1^2(2s)$. for $a = 0, 0.05, 0.1, \dots, 2.05$

Product structure for Two-Speed TASEP

Theorem (At the F_1–F_1 shock, Ferrari, Nej. '13)
Let $x_n(0) = -2n$ for $n \in \mathbb{Z}$. For $\alpha < 1$ let $\eta = \frac{2-\alpha}{4}$ and $v = -\frac{1-\alpha}{2}$. Then it holds

$$\lim_{t\to\infty} \mathbb{P}\left(\frac{x_{\eta t+\xi t^{1/3}}(t) - vt}{t^{1/3}} \leq s\right) = F_1\left(\frac{s-2\xi}{\sigma_1}\right) F_1\left(\frac{s-\frac{2\xi}{2-\alpha}}{\sigma_2}\right),$$

where F_1 is the GOE Tracy-Widom distribution, $\sigma_1 = \frac{1}{2}$ and $\sigma_2 = \frac{\alpha^{1/3}(2-2\alpha+\alpha^2)^{1/3}}{2(2-\alpha)^{2/3}}$.

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One recovers GOE by changing $s \to s + 2\xi$ and $\xi \to +\infty$, resp. by $s \to s + 2\xi/(2-\alpha)$ and $\xi \to -\infty$

Outline

Last Passage Percolation (LPP)

Ansatz: Reformulate the problem in terms of a **generic LPP model:** Let $(\omega_{i,j})_{(i,j)\in\mathbb{Z}^2}$ be independent random variables, $\mathcal{L} \subseteq \mathbb{Z}^2$ and π be an up-right path from $\mathcal{L}$ to (m,n). Then $L_{\mathcal{L}\to(m,n)}$ is the maximal percolation time

$$L_{\mathcal{L}\to(m,n)} := \max_{\pi:\mathcal{L}\to(m,n)} \sum_{(i,j)\in\pi} \omega_{i,j} = \sum_{(i,j)\in\pi^{\max}} \omega_{i,j}$$

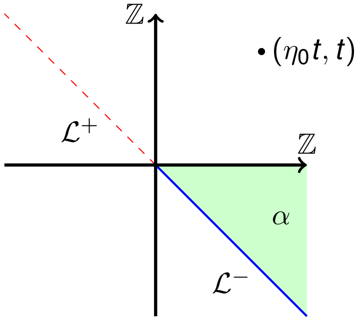
TASEP with IC $(x_k(0))_{k\in\mathbb{Z}}$. Setting

- $\omega_{i,j}$ to be the time particle j needs to jump from site $i-j-1$ to $i-j$,
- $\mathcal{L} = \{(k,u) | u = k + x_k(0), k \in \mathbb{Z}\}$,

it holds

$$\mathbb{P}\left(L_{\mathcal{L}\to(m,n)} \leq t\right) = \mathbb{P}\left(x_n(t) \geq m-n\right).$$

Example: Two-Speed TASEP as LPP



- $\mathcal{L} = \{(u, -u) : u \in \mathbb{Z}\} = \mathcal{L}^+ \cup \mathcal{L}^-$
- $\omega_{i,j} \sim \exp(1)$ in white region, $\exp(\alpha)$ in green.

Strategy

- write $\mathcal{L} = \mathcal{L}^+ \cup \mathcal{L}^-$, with $\mathcal{L}^+ \subseteq \{(x, y) : x \leq 0, y \geq 0\}$, and $\mathcal{L}^- \subseteq \{(x, y) : x \geq 0, y \leq 0\}$,

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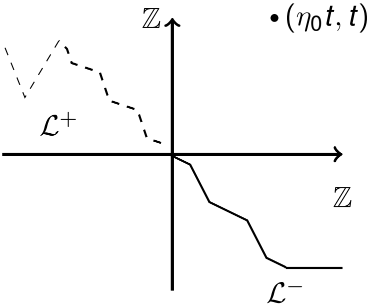
Remarks:

- asymptotic independence is equivalent to asymptotic non-intersection of the maximizing paths
- we will show that π_-^{max} of $L_{\mathcal{L}_- \to (m,n)}$ and π_+^{max} of $L_{\mathcal{L}_+ \to (m^+, n^+)}$ intersect with vanishing probability

Generic Theorem



Assume that there exists some μ such that

$$\lim_{t\to\infty} \mathbb{P}\left(\frac{L_{\mathcal{L}^+\to(\eta_0 t,t)} - \mu t}{t^{1/3}} \leq s\right) = G_1(s),$$

$$\lim_{t\to\infty} \mathbb{P}\left(\frac{L_{\mathcal{L}^-\to(\eta_0 t,t)} - \mu t}{t^{1/3}} \leq s\right) = G_2(s).$$

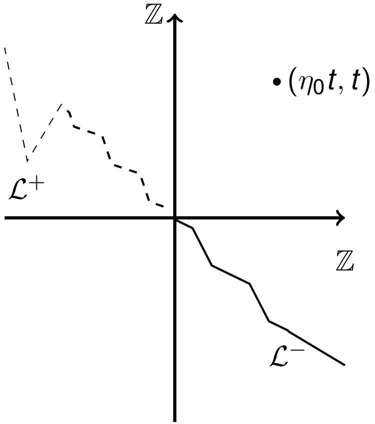
Theorem (Ferrari, Nej. '13)

Under some assumptions we have

$$\lim_{t\to\infty} \mathbb{P}\left(\frac{L_{\mathcal{L}\to(\eta_0 t,t)} - \mu t}{t^{1/3}} \leq s\right) = G_1(s)G_2(s),$$

where $\mathcal{L} = \mathcal{L}^+ \cup \mathcal{L}^-$.

On the assumptions



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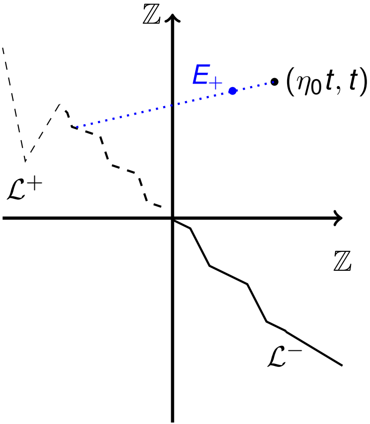
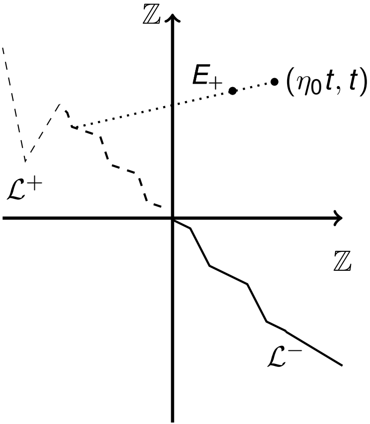


I. Assume that we have a point $E^+ = (\eta_0 t - \kappa t^\nu, t - t^\nu)$ such that for some μ_0, and $\nu \in (1/3, 1)$ it holds

$$\frac{L_{\mathcal{L}^+ \to E_+} - \mu t + \mu_0 t^\nu}{t^{1/3}} \to G_1$$

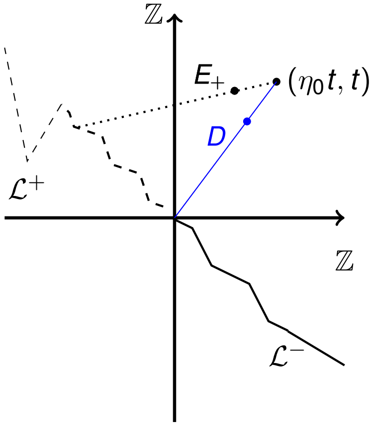
$$\frac{L_{E^+ \to (\eta_0 t, t)} - \mu_0 t^\nu}{t^{\nu/3}} \to G_0,$$

On the assumptions



I. *Slow Decorrelation*

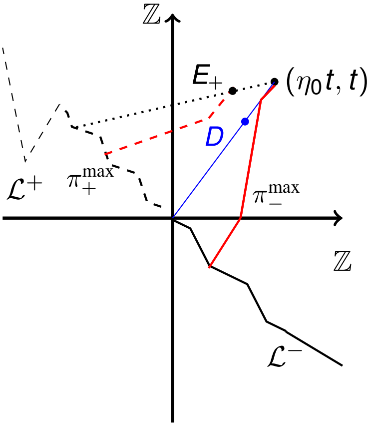
On the assumptions



I. *Slow Decorrelation*

II. Assume there is a point D on $\overline{(0,0)(\eta_0 t, t)}$ and to the right of E_+ such that $\pi_+^{\max}$ and $\pi_-^{\max}$ cross $\overline{(0,0)D}$ with vanishing probability.

On the assumptions



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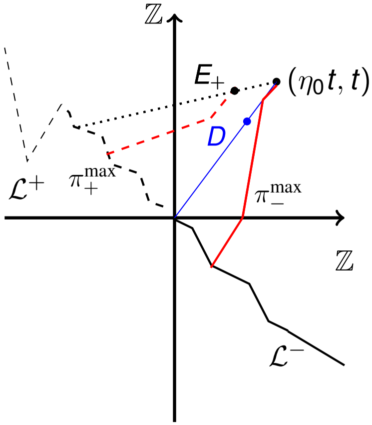


I. *Slow Decorrelation*

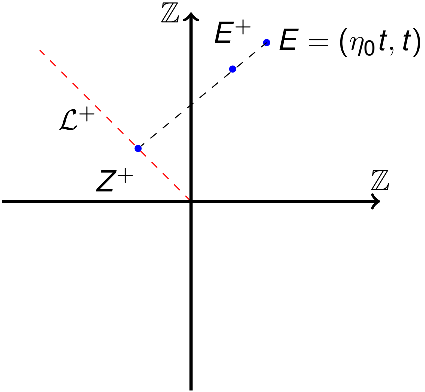
II. *No crossing*

Some remarks:

- (I.) is related to the universal phenomenon known as slow decorrelation [CFP '12]
- (II.) follows if we have that the 'characteristic lines' of the two LPP problems meet at $(\eta_0 t, t)$, together with the transversal fluctuations which are only $\mathcal{O}(t^{2/3})$ [Johansson'00]

Slow decorrelation for Two-Speed TASEP

- E^+ lies on $\overline{Z^+E}$, where Z^+ is the orthogonal projection of E on $\mathcal{L}^+$.
- Z^+ satisfies $\mu_{Z^+\to E} = \mu_{\mathcal{L}^+\to E} = \frac{4}{2-\alpha}$ where $\mu_{Z^+\to E}$ is s.t. $\frac{L_{Z^+\to E} - \mu_{Z^+\to E}t}{t^{1/3}}$ has non-trivial limit (leading order term).



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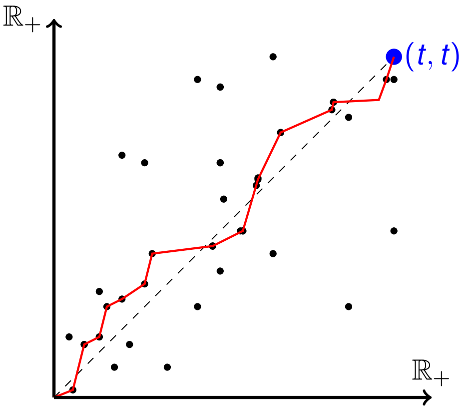
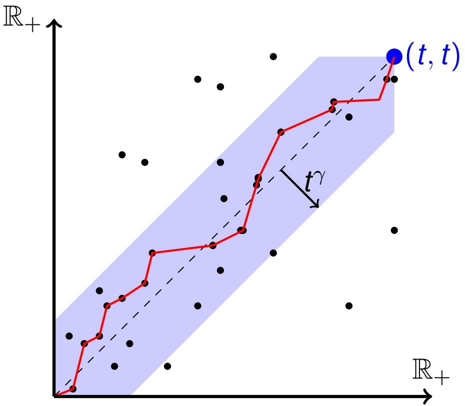


Consider a Poisson Point Process on $\mathbb{R}_+^2$ with intensity one. The length $\ell(\pi)$ of a path π is the number of Poisson Points on it.

$$L_{(0,0)\to(t,t)} = \max_{\substack{\pi:(0,0)\to(t,t) \\ \text{north-east}}} \ell(\pi)$$
$$= \ell(\pi^{\max})$$



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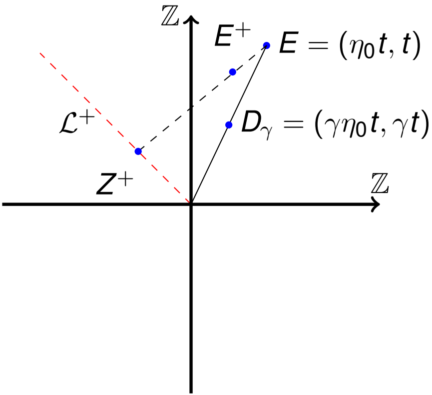
Theorem (Johansson '00)

For $A_t^\gamma = \{(x,y) \in [0,t]^2 : -\sqrt{2}t^\gamma \le -x+y \le \sqrt{2}t^\gamma\}$ *we have for any* $\gamma > 2/3$

$$\lim_{t\to\infty} \mathbb{P}(\pi^{\max} \subseteq A_t^\gamma) = 1.$$

Transversal Fluctuations



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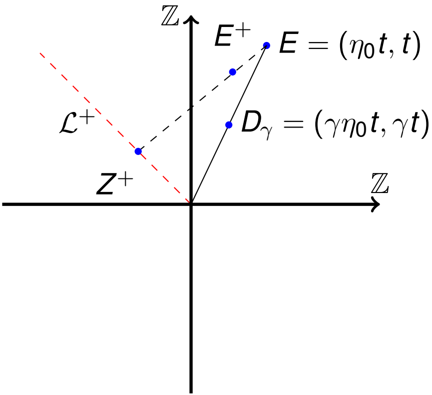
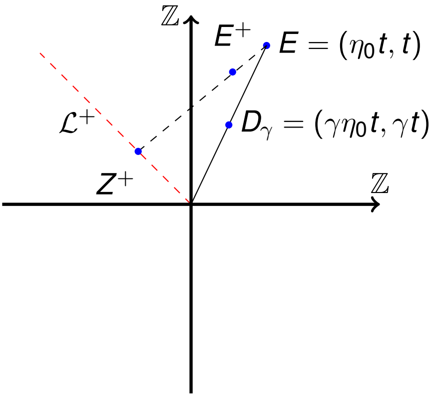


$\gamma \in [0, 1 - t^{\beta-1}], \beta \in (1/3, 1]$

$\varepsilon = C_1 t^{\beta-1}$

Transversal Fluctuations

Let $\mu_{\mathcal{L}^+\to E^+}, \mu_{\mathcal{L}^+\to D_\gamma}$ and $\mu_{D_\gamma\to E^+}$, be the leading order terms of $L_{\mathcal{L}^+\to E^+}, L_{\mathcal{L}^+\to D_\gamma}$ and $L_{D_\gamma\to E^+}$.



$\gamma \in [0, 1 - t^{\beta-1}], \beta \in (1/3, 1]$

$\varepsilon = C_1 t^{\beta-1}$

$$E_{D_\gamma} = \{L_{\mathcal{L}^+\to D_\gamma} > (\mu_{\mathcal{L}^+\to D_\gamma} + \varepsilon/2)t\} \cup \{L_{D_\gamma\to E^+} > (\mu_{D_\gamma\to E^+} + \varepsilon/2)t\}$$

- I. $\mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right) \le C\exp(-ct^{\beta-1/3}) \qquad (t > t_0)$
  This result is based on translating the $L_{\mathcal{L}^+\to E^+}$ LPP into TASEP, and the decay of the corresponding kernel K
- II. We have

$$\frac{(\mu_{\mathcal{L}^+\to D_\gamma} + \mu_{D_\gamma\to E^+} + \varepsilon - \mu_{\mathcal{L}^+\to E^+})t}{t^{1/3}} \le -Ct^{\beta-1/3}$$

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Let $I_{D_\gamma} = \{D_\gamma \in \pi^{\max}\}$. We can conclude

$$\mathbb{P}(I_{D_\gamma}) \leq \mathbb{P}\left(I_{D_\gamma} \cap (\bigcap E^c_{D_\gamma})\right) + \mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right)$$
$$\leq \mathbb{P}(L_{\mathcal{L}^+\to E^+} \leq \mu_{\mathcal{L}^+\to E^+} t - Ct^\beta) + C\exp(-ct^{\beta-1/3})$$

- I. $\mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right) \leq C\exp(-ct^{\beta-1/3}) \qquad (t > t_0)$
  This result is based on translating the $L_{\mathcal{L}^+\to E^+}$ LPP into TASEP, and the decay of the corresponding kernel K
- II. We have

$$\frac{(\mu_{\mathcal{L}^+\to D_\gamma} + \mu_{D_\gamma\to E^+} + \varepsilon - \mu_{\mathcal{L}^+\to E^+})t}{t^{1/3}} \leq -Ct^{\beta-1/3}$$

Let $I_{D_\gamma} = \{D_\gamma \in \pi^{\max}\}$. We can conclude

$$\begin{aligned}\mathbb{P}(I_{D_\gamma}) &\leq \mathbb{P}\left(I_{D_\gamma} \cap (\bigcap E^c_{D_\gamma})\right) + \mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right)\\ &\leq \mathbb{P}(L_{\mathcal{L}^+\to E^+} \leq \mu_{\mathcal{L}^+\to E^+}t - Ct^\beta) + C\exp(-ct^{\beta-1/3})\\ &\leq \mathbb{P}(L_{Z^+\to E^+} \leq \mu_{\mathcal{L}^+\to E^+}t - Ct^\beta) + C\exp(-ct^{\beta-1/3})\end{aligned}$$

- I. $\mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right) \le C\exp(-ct^{\beta-1/3}) \qquad (t > t_0)$
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- I. $\mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right) \leq C\exp(-ct^{\beta-1/3}) \qquad (t > t_0)$
  This result is based on translating the $L_{\mathcal{L}^+\to E^+}$ LPP into TASEP, and the decay of the corresponding kernel K
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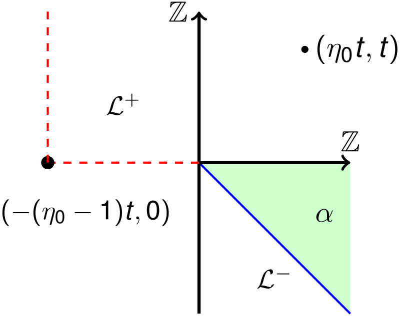
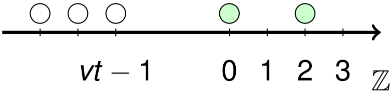
Let $I_{D_\gamma} = \{D_\gamma \in \pi^{\max}\}$. We can conclude

$$\begin{aligned}
\mathbb{P}(I_{D_\gamma}) &\leq \mathbb{P}\left(I_{D_\gamma} \cap (\bigcap E^c_{D_\gamma})\right) + \mathbb{P}\left(\bigcup_\gamma E_{D_\gamma}\right) \\
&\leq \mathbb{P}(L_{\mathcal{L}^+\to E^+} \leq \mu_{\mathcal{L}^+\to E^+}t - Ct^\beta) + C\exp(-ct^{\beta-1/3}) \\
&\leq \mathbb{P}(L_{Z^+\to E^+} \leq \mu_{\mathcal{L}^+\to E^+}t - Ct^\beta) + C\exp(-ct^{\beta-1/3}) \\
&\leq \tilde{C}\exp(-\tilde{c}t^{\beta-1/3})
\end{aligned}$$

This implies $\mathbb{P}(\bigcup_\gamma I_{D_\gamma}) \leq t\tilde{C}\exp(-\tilde{c}t^{\beta_0-1/3}) \to 0$, since only $\mathcal{O}(t)$ many points D_γ.

Outline





$\eta_0 = \frac{\alpha(3-2\alpha)}{2-\alpha}$, $\omega_{i,j} \sim \exp(1)$ in white region, $\exp(\alpha)$ in green.

Particles initially occupy $2\mathbb{N}_0 \cup \{-vt-1, -vt-2, \ldots\}$, where $v = \frac{(1-\alpha)^2}{2(2-\alpha)}$.

Theorem (At the F_2–F_1 shock, Ferrari, Nej' 13)

For $\alpha < 1$ let $\mu = 4$ and $v = -\frac{(1-\alpha)^2}{2(2-\alpha)}$. Let $x_n(0) = vt - n$ for $n \geq 1$ and $x_n(0) = -2n$ for $n \leq 0$. Then it holds

$$\lim_{t\to\infty} \mathbb{P}\left(x_{t/\mu+\xi t^{1/3}}(t) \geq vt - st^{1/3}\right) = F_2\left(\frac{s - c_1\xi}{\sigma_1}\right) \times F_1\left(\frac{s - c_2\xi}{\sigma_2}\right),$$

with $c_1, c_2, \sigma_1, \sigma_2$ some constants depending on α. F_2 is the GUE Tracy-Widom distribution from random matrix theory.

Thanks for your attention!

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