

Simpson-Regel.

Hermite Basis: H0,H1,H2,H3:

$$p(x)=f(\alpha) H_0(x)+f(\alpha+h/2) H_1(x) + f'(\alpha+h/2) H_2(x) + f(\alpha+h) H_3(x)$$

> $H_0 := -4/h^3 * (x-\alpha-h/2)^2 * (x-\alpha-h);$

$$H_0 := -\frac{4 \left(x - \alpha - \frac{h}{2}\right)^2 (x - \alpha - h)}{h^3}$$

> $H_1 := -(x-\alpha) * (x-\alpha-h) * 4/h^2;$

$$H_1 := -\frac{4 (x - \alpha) (x - \alpha - h)}{h^2}$$

> $H_2 := -(x-\alpha) * (x-\alpha-h) * (x-\alpha-h/2) * 4/h^2;$

$$H_2 := -\frac{4 (x - \alpha) (x - \alpha - h) \left(x - \alpha - \frac{h}{2}\right)}{h^2}$$

> $H_3 := (x-\alpha-h/2)^2 * (x-\alpha) * 4/h^3;$

$$H_3 := \frac{4 \left(x - \alpha - \frac{h}{2}\right)^2 (x - \alpha)}{h^3}$$

Koeffizienten

> $\text{simplify}(\text{int}(H_0, x=\alpha..alpha+h));$

$$\frac{h}{6}$$

> $\text{simplify}(\text{int}(H_1, x=\alpha..alpha+h));$

$$\frac{2h}{3}$$

> $\text{simplify}(\text{int}(H_2, x=\alpha..alpha+h));$

$$0$$

> $\text{simplify}(\text{int}(H_3, x=\alpha..alpha+h));$

$$\frac{h}{6}$$

Lagrange Basis: L0,L1,L2:

$p(x)=f(\alpha) L_0(x)+f(\alpha+h/2) L_1(x) + f(\alpha+h) L_2(x)$

> `L0:=(x-alpha-h/2)*(x-alpha-h)*2/h^2;`

$$L_0 := \frac{2 \left(x - \alpha - \frac{h}{2} \right) (x - \alpha - h)}{h^2}$$

> `L1:=- (x-alpha)*(x-alpha-h)*4/h^2;`

$$L_1 := - \frac{4 (x - \alpha) (x - \alpha - h)}{h^2}$$

> `L2:=(x-alpha)*(x-alpha-h/2)*2/h^2;`

$$L_2 := \frac{2 (x - \alpha) \left(x - \alpha - \frac{h}{2} \right)}{h^2}$$

Koeffizienten

> `simplify(int(L0,x=alpha..alpha+h));`

$$\frac{h}{6}$$

> `simplify(int(L1,x=alpha..alpha+h));`

$$\frac{2 h}{3}$$

> `simplify(int(L2,x=alpha..alpha+h));`

$$\frac{h}{6}$$

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