Stochastic Analysis and Heat Kernels on Manifolds

This seminar gives an introduction to Stochastic Analysis on Manifolds. Topics include:

- Martingales on manifolds, diffusion processes and stochastic differential equations, which can symbolically be written as $dX_t = V_\alpha(X_t) \circ dZ_\alpha^t$. In particular, existence and uniqueness for such SDE’s, connection with smooth second order elliptic operators on manifolds. In addition, the horizontal lift and stochastic development, two concepts central to the Eells-Elworth-Malliavin construction of Brownian motions on a Riemannian manifold, will be introduced.

- Brownian motion on a Riemannian manifold as a diffusion generated by the Laplace-Beltrami-operator; the effect of curvature on the radial process of Brownian motion by comparing it with the same process on a radially symmetric manifolds.

- The heat kernel on manifolds and its connections with the Brownian Motion; in addition, the short-time behaviour of the heat kernel and the Brownian Motion, especially Varadhans asymptotic relation, and the derivatives of the logarithmic heat kernel.

- Li-Yau-type Harnack inequalities and Gaussian estimates for the heat equation on manifolds, comparison theorems.

The seminar will start in the second week of November.

Literature

- E.P. Hsu *Stochastic analysis on Manifolds*
- N. Ikeda, S. Watanabe *Stochastic Differential equations and Diffusion processes*
- E.B. Davies *Heat kernels and spectral theory*
- A. Grigor’yan *Heat kernel and Analysis on Manifolds*

Required knowledge: Basic Stochastic Analysis, Basic Differential Geometry.

Preliminary discussion: Tue, 19.10.2010, 16.15, room 4.050, LWK.
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