Universität Bonn
Sommersemester 2025

Institut für Angewandte Mathematik
Johannes Alt

# Graduate seminar on Probability Theory: Topics in Random Matrix Theory

**Tuesdays at 16 (c.t.) (duration: 90 minutes)**
**Room N 0.003**

## List of Talks

| No. | Date | Title and references | Speaker |
|---|---|---|---|
| 0 | April 8 | **Introduction**<br>• Simulations<br>• Eigenvalues, singular values, Weyl's inequality<br>• Some basics about Schwartz functions (on $\mathbb{C}$) and (tempered) distributions | Johannes Alt |
| 1 | April 15 | **Wigner's semicircle law**<br>Define empirical eigenvalue measure (or empirical spectral measure), state Wigner's semicircle law (without proof!): [Kem22, Theorem 2.3] (or [AGZ10, Theorem 2.1.1]).<br>Prove the convergence of moments in expectation [Kem22, Proposition 4.1, Section 4.1] (alternatively: [AGZ10, Lemma 2.1.6 and its proof] and its prerequisites).<br>*Optional:* Proof sketch of Wigner's semicircle law in expectation. | D.M. |
| 2 | April 22 | **Matrices with iid entries and the circular law**<br>Define empirical eigenvalue measure, empirical singular value measure and weak convergence of measures (see [BC12, Section 1]).<br>[BC12, Example 1.2]. Define tightness of family of probability measures.<br>[BC12, Section 2] without historical comments. [BC12, Lemma 3.1] and its proof. | R.U. |
| 3 | April 29 | **Circular law for complex Ginibre matrix**<br>[BC12, Section 3] until the end of the proof of Theorem 3.5 without Lemma 3.1 and its proof.<br>In particular, define *Complex Ginibre Ensemble (matrix)* and conclude its diagonalisability. Add more details from [Meh04] in the proof of Theorem 3.4.<br>*Optional:* more details in the proof of Theorem 3.5 from [Hwa86]. | D.W. |
| 4 | May 6 | **Logarithmic potential and Hermitization**<br>[BC12, Section 4.1] without Remarks 4.4, 4.6, 4.7 and 4.8. | P.A. |
| 5 | May 13 | **Circular law for general iid matrices**<br>Proof of [BC12, Theorem 2.2] for general matrices with iid entries (iid matrices): [BC12, Section 4.2].<br>Proofs of Lemmata 4.11, 4.13 and 4.14: [BC12, Section 4.3]. | T.M. |

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| 6 | May 20 | **Convergence of singular value measures**<br>Proof of Corollary 4.10: [BC12, Section 4.5] | H.S. |
| 7 | May 27 | **Smallest singular value – Part 1**<br>Proof of [BC12, Lemma 4.12] with bounded density assumption in [BC12, Section 4.4].<br>[BC12, Appendix A] until the end of the proof of Lemma A.2, in particular, statement of Lemma A.1, overview of its proof. | J.K. |
| 8 | June 3 | **Smallest singular value – Part 2**<br>[BC12, Appendix A] starting after the proof of Lemma A.2.<br>*Optional:* Proof sketch of circular law in probability using [BC12, Lemma A.1] | J.S. |
| 9 | June 17 | **Fluctuation of spectral radius of complex Ginibre matrix**<br>Theorem 1 of [Rid03] and its proof.<br>(Compare also [BC12, Theorems 3.6 and 3.7]. A part of [BC12, Theorem 3.6] is proved inside [Rid03, proof of Theorem 1].) | J.L. |
| 10 | June 24 | **Convergence of spectral radius for iid matrices**<br>Theorems 1.1 and 1.2 of [BCGZ22] and their proofs. | S.W. |

## References and how to access them

The references [AGZ10] and [Kem22] are freely and legally available via the given links. All other references listed below (with the exception of [Hwa86]) can be accessed digitally from the network of the University of Bonn. For [BC12], [BCGZ22] and [Rid03], the links should work. The books [Meh04] and [For10] can be accessed digitally via the webpage of the Bonn University library at https://www.ulb.uni-bonn.de/en. The books [AGZ10], [Meh04], [Hwa86] or [For10] can be found in the library.

If you experience difficulties to access a reference then please contact Johannes Alt via email.

## References

[AGZ10] G.W. Anderson, A. Guionnet, and O. Zeitouni, *An introduction to random matrices*, Cambridge Studies in Advanced Mathematics, vol. 118, Cambridge University Press, Cambridge, 2010, Available at https://www.wisdom.weizmann.ac.il/~zeitouni/cupbook.pdf; see also Errata Sheet at https://www.wisdom.weizmann.ac.il/~zeitouni/cormat.pdf.

[BC12] C. Bordenave and D. Chafaï, *Around the circular law*, Probab. Surv. **9** (2012), 1–89, Available at https://doi.org/10.1214/11-PS183.

[BCGZ22] C. Bordenave, D. Chafaï, and D. García-Zelada, *Convergence of the spectral radius of a random matrix through its characteristic polynomial*, Probab. Theory Related Fields **182** (2022), no. 3-4, 1163–1181, Available at https://doi.org/10.1007/s00440-021-01079-9.

[For10] P. J. Forrester, *Log-gases and random matrices*, London Mathematical Society Monographs Series, vol. 34, Princeton University Press, Princeton, NJ, 2010. MR 2641363

[Hwa86] C.-R. Hwang, *A brief survey on the spectral radius and the spectral distribution of large random matrices with i.i.d. entries*, Random matrices and their applications (Brunswick, Maine, 1984), Contemp. Math., vol. 50, Amer. Math. Soc., Providence, RI, 1986, pp. 145–152. MR 841088

[Kem22] T. Kemp, *Introduction to random matrix theory*, 2022, Lecture notes, available at https://mathweb.ucsd.edu/~tkemp/RMT.Notes.pdf.

[Meh04] M.L. Mehta, *Random matrices*, third ed., Pure and Applied Mathematics (Amsterdam), vol. 142, Elsevier/Academic Press, Amsterdam, 2004. MR 2129906

[Rid03] B. Rider, *A limit theorem at the edge of a non-Hermitian random matrix ensemble*, J. Phys. A **36** (2003), no. 12, 3401–3409, Available at https://doi.org/10.1088/0305-4470/36/12/331.