Universität Bonn
Wintersemester 2023/2024

Institut für Angewandte Mathematik
Johannes Alt

Graduate seminar on Interacting Random Systems: Selected topics in random matrices and random operators

Wednesdays at 12 (c.t.) (duration: 90 minutes)
Room N 0.007

List of Talks

No.	Date	Title and references	Speaker
1	11.10.23	**Spectral theory for bounded operators on a Hilbert space** Introduction/Recap about: • spectral theorem for self-adjoint bounded operators on Hilbert space • spectral calculus, spectral measure • spectral types • Stone's theorem, solution to Schrödinger equation	Johannes Alt
2	25.10.23	**Anderson model and definition of localization** Section 2 of [Sto11]: definition of Anderson model and basic properties parts of Section 3 of [Sto11]: definitions of spectral localization and dynamical localization, expectations for the Anderson model in different dimensions (see also Figure 1.2 in [AW15])	P. v. G.
3	8.11.23	**Decay of fractional moments of Green function at large disorder** Section 4 of [Sto11] including proofs of Lemmas 4.1 and 4.2 (proof of Lemma 4.2 is not presented in [Sto11], but see e.g. [Gra94, Lemma 6])	C. N.
4	15.11.23	**Localization from decay of fractional moments** Section 5 of [Sto11] + state RAGE-Theorem needed here (see e.g. [AW15, Theorem 2.6])	J. D. F.
5	22.11.23	**Finite volume methods** Section 6 + (necessary parts of) Appendix A of [Sto11]	P. Z.
6	29.11.23	**Integrated density of states and Wegner estimate** Definitions of the density of states and the integrated density of states (IDS): [Kir08, Definition 5.4] (possibly some additional explanations from [Kir08, Section 5.1]) Wegner estimate and continuity of IDS [Kir08, Section 5.5 until Theorem 5.27] (including the necessary parts from [Kir08, Section 5.4])	D. H.
7	13.12.23	**Band edge localization** Section 7 of [Sto11] + statement of Combes-Thomas estimate (see e.g. [Kir08, Theorem 11.2]) + missing proof of [Sto11, Lemma 7.3] (see e.g. [ASFH01, Lemma 2.3])	F. A.

(continued on the next page)

No.	Date	Title and references	Speaker
8	20.12.23	**Random walk representation** Section 4 of [Hun08] (some motivation/intuition explained in Section 3 of [Hun08] might be helpful)	Y. T.
9	10.01.24	**Localization at smaller disorder via random walk representation** Theorem 1 of [Sch15], the remark following Theorem 1 and the proof of Theorem 1 in [Sch15] (as well as necessary definitions and helpful explanations that might come earlier in [Sch15])	Johannes Alt

Mathematical comments

- We will always assume that the random variables defining V are bounded. Thus, H_λ will be a bounded linear operator on $\ell^2(\mathbb{Z}^d)$. This avoids considering the domain of definition of H_λ and, thus, simplifies many statements and arguments.
- Note that [Sto11] has a slightly different convention/definition, compared to the other references, for the discrete Laplacian and, hence, the Anderson model. Therefore, the spectrum of both operators is shifted by $-2d$ compared to the other references.

References

[ASFH01] Michael Aizenman, Jeffrey H. Schenker, Roland M. Friedrich, and Dirk Hundertmark, *Finite-volume fractional-moment criteria for Anderson localization*, vol. 224, 2001, Dedicated to Joel L. Lebowitz, pp. 219–253. MR 1868998

[AW15] Michael Aizenman and Simone Warzel, *Random operators*, Graduate Studies in Mathematics, vol. 168, American Mathematical Society, Providence, RI, 2015, Disorder effects on quantum spectra and dynamics. MR 3364516

[Gra94] Gian Michele Graf, *Anderson localization and the space-time characteristic of continuum states*, J. Statist. Phys. **75** (1994), no. 1-2, 337–346. MR 1273061

[Hun08] Dirk Hundertmark, *A short introduction to Anderson localization*, Analysis and stochastics of growth processes and interface models, Oxford Univ. Press, Oxford, 2008, pp. 194–218. MR 2603225, Preprint available at https://faculty.math.illinois.edu/~dirk/preprints/localization3.pdf

[Kir08] Werner Kirsch, *An invitation to random Schrödinger operators*, Random Schrödinger operators, Panor. Synthèses, vol. 25, Soc. Math. France, Paris, 2008, With an appendix by Frédéric Klopp, pp. 1–119. MR 2509110, Preprint available at https://arxiv.org/pdf/0709.3707.pdf

[Sch15] Jeffrey Schenker, *How large is large? Estimating the critical disorder for the Anderson model*, Lett. Math. Phys. **105** (2015), no. 1, 1–9. MR 3294429

[Sto11] Günter Stolz, *An introduction to the mathematics of Anderson localization*, Entropy and the quantum II, Contemp. Math., vol. 552, Amer. Math. Soc., Providence, RI, 2011, pp. 71–108. MR 2868042, Preprint available at https://arxiv.org/pdf/1104.2317