Universität Bonn
Sommersemester 2026

Institut für Angewandte Mathematik
Johannes Alt

Graduate seminar on Probability Theory: Log gases and random matrices

---

Contents

The goal of this seminar is to study N random variables $(\lambda_1, \ldots, \lambda_N)$ on the real line with the distribution

$$\mathbb{P}\big((\lambda_1, \ldots, \lambda_N) \in A\big) = \int_A Z_{N,\beta}^{-1} \,\mathrm{e}^{-\beta N \sum_{i=1}^N V(x_i)} \left(\prod_{i<j} |x_i - x_j|^\beta \right) \mathrm{d}x_1 \ldots \mathrm{d}x_N \qquad (1)$$

for any Borel-measurable $A \subset \mathbb{R}^N$. Here, $\beta > 0$ is a constant, $V : \mathbb{R} \to \mathbb{R}$ is a nice function and $Z_{N,\beta}$ is a normalization constant. The random variables $(\lambda_1, \ldots, \lambda_N)$ with the joint density (1) are called β-*ensemble*. If $\beta = 1$ and $\beta = 2$ and V is quadratic, then the eigenvalues of the Gaussian orthogonal and Gaussian unitary ensemble, respectively, have this joint density. These are the most important examples of Hermitian random matrices.

In the seminar, we will first establish this connection as well as a connection to tridiagonal random matrices. Then we will show that the empirical measure $\frac{1}{N}\sum_{i=1}^N \delta_{\lambda_i}$ has a deterministic limit for $N \to \infty$ and study its large deviations. That is, how likely it is for large N that the empirical limit differs substantially from its limit. We will also answer the analogous large deviation question for the maximum of $\lambda_1, \ldots, \lambda_N$. Depending on the number of participants, we will afterwards study the scaling limit of the Gaussian unitary ensemble in the bulk or the relation between β-ensembles and Dyson-Brownian motion.

From the point of view of physics, the density (1) describes the statistics of a gas of N particle on the real line that are subject to a logarithmic interaction and a potential V.

Preliminary meeting

- Tuesday, January 27, 2026, 5:00 pm, room 2.040, Endenicher Allee 60
- If you are interested but missed the preliminary meeting then write an email to Johannes Alt as soon as possible. The topics will be distributed in the first half of February. After that, potentially remaining slots can still be filled.

Prerequisites

Measure theory and measure-theoretic probability theory. For some talks, some knowledge in complex analysis might be helpful. If we cover the part about Dyson-Brownian motion then these talks will require some basic knowledge in stochastic analysis.

Literature

The seminar will mostly follow the monograph [AGZ10], in particular, Sections 2.6 and 4.5.1. For some talks, we will also use the lecture notes [Kem22]. Alternative monographs are [For10] and [PS11].

The book [For10] can be accessed digitally via the webpage of the Bonn University library at https://www.ulb.uni-bonn.de/en. The monographs [AGZ10], and [For10] can be found in the library.

References

[AGZ10] G.W. Anderson, A. Guionnet, and O. Zeitouni, *An introduction to random matrices*, Cambridge Studies in Advanced Mathematics, vol. 118, Cambridge University Press, Cambridge, 2010, Available at https://www.wisdom.weizmann.ac.il/~zeitouni/cupbook.pdf; see also Errata Sheet at https://www.wisdom.weizmann.ac.il/~zeitouni/cormat.pdf.

[For10] P. J. Forrester, *Log-gases and random matrices*, London Mathematical Society Monographs Series, vol. 34, Princeton University Press, Princeton, NJ, 2010. MR 2641363

[Kem22] T. Kemp, *Introduction to random matrix theory*, 2022, Lecture notes, available at https://mathweb.ucsd.edu/~tkemp/RMT.Notes.pdf.

[PS11] L. Pastur and M. Shcherbina, *Eigenvalue distribution of large random matrices*, Mathematical Surveys and Monographs, vol. 171, American Mathematical Society, Providence, RI, 2011. MR 2808038