

Sheet 11, “Stochastic Analysis”

For discussion in the tutorials

Problem 1 (Tightness of a sequence of càdlàg random variables)

Let $\{X_i\}_{i \in \mathbb{N}}$ be i.i.d. random variables with support on $\mathbb{R}_{\geq 0}$ and assume that

$$n\mathbb{P}\left[X_1 > nx\right] \rightarrow cx^{-1} \quad \text{as } n \rightarrow \infty,$$

for some $c > 0$. Theorem 6.12 in the lecture notes states that

$$S_n(t) := \frac{1}{cn \ln(n)} \sum_{i=1}^{\lfloor nt \rfloor} X_i \rightarrow t \quad \text{as } n \rightarrow \infty,$$

in law with respect to the Skorokhod J_1 -topology. Complete the proof by showing that the sequence S_n is tight in the J_1 -topology.

Hint: Mimic the tightness proof of Theorem 6.5. The present case is even easier since $S_n(t) \rightarrow t$ for every fixed $t \geq 0$.