Sheet 11, "Stochastic Analysis"

To be discussed on July 07, 2021

Problem 1 (Tightness of a sequence of càdlàg random variables)

Let $\{X_i\}_{i \in \mathbb{N}}$ be i.i.d. random variables with support on $\mathbb{R}_{\geq 0}$ and assume that

$$n\mathbb{P}\Big[X_1 > nx\Big] \to cx^{-1} \qquad\qquad \text{as } n \to \infty,$$

for some $c > 0$. Theorem 7.12 in the lecture notes states that

$$S_n(t) := \frac{1}{cn \ln(n)} \sum_{i=1}^{[nt]} X_i \to t \qquad\qquad \text{as } n \to \infty,$$

in law with respect to the Skorokhod J_1-topology. Complete the proof by showing that the sequence S_n is tight in the J_1-topology.

Hint: Mimic the tightness proof of Theorem 7.5. However, the present case is in fact easier: use that for every fixed $t \geq 0$: $S_n(t) \to t$.