Institute for Applied Mathematics
SS 2022
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Stochastic Processes
Sheet 2

To hand in via ecampus before Friday, April 15

New tutorial class: There will be an additional tutorial class on Monday, 10ct, room TBA. You can still register for the exercise classes over the weekend (or maybe change, just make sure you are registered for exactly one tutorial). You must hand in your solutions in **FIXED** groups of three, and are free to form groups with students from different tutorials. Write the complete names and email-addresses of all three students on each hand-in, and hand in your solutions to one tutor of your choice. We will distribute the workload among the tutors.

Exercise 1 *[5 Pkt]*

Find the open, closed and compact subsets of the metric space $(\mathbb{Z}^d, \rho)$, where $\mathbb{Z}$ is the set of integer numbers and ρ is the euclidean metric in $\mathbb{R}^d$. Define the corresponding Borel-σ-algebra $\mathcal{B}(\mathbb{Z}^d)$.

Exercise 2 *[5 Pkt]*

Let $\{(X_n, \mathcal{A}_n, \mu_n)\}_{n\in\mathbb{N}}$ be a family of measure spaces, where the sets X_n are pairwise disjoints. We define the measure space $(X, \mathcal{A}, \mu)$, where $X = \bigcup_n X_n$,

$$\mathcal{A} = \{B : B \cap X_n \in \mathcal{A}_n \text{ for all } n\}$$

and $\mu(B) = \sum_n \mu_n(B \cap X_n)$. Show that

1. $\mathcal{A}$ is a σ-algebra;
2. μ is a measure;
3. μ is σ-finite if and only if all μ_n are σ-finite.

Exercise 3 *[5 Pkt]*

Let $(X, \mathcal{A}, \mu)$ be a measure space. Let $A_1 \triangle A_2 = (A_1 \setminus A_2) \cup (A_2 \setminus A_1)$ for $A_1, A_2 \in \mathcal{A}$. Show that:

1. If $A_1, A_2 \in \mathcal{A}$ and $\mu(A_1 \triangle A_2) = 0$, then $\mu(A_1) = \mu(A_2)$.

2. If the measure space is complete, $A_1, A_1 \triangle A_2 \in \mathcal{A}$ and $\mu(A_1 \triangle A_2) = 0$, then we have that $A_2 \in \mathcal{A}$.

Remark: A measure space $(X, \mathcal{A}, \mu)$ is complete if the σ-algebra $\mathcal{A}$ contains all the subsets of μ-null sets, i.e., if $B \in \mathcal{A}$, $\mu(B) = 0$ and $A \subset B$ then $A \in \mathcal{A}$.

Tea Time With Women in Mathematics, April 29: All female, intersexual, non-binary, transgender and agender people - from students to professors - are invited to network over a cup of tea. The event takes place from 4pm to 6pm at the Zeichensaal at the Wegelerstrasse 10. For more information, write an email to teatime@hausdorff-center.uni-bonn.de or check the homepage.