Institute for Applied Mathematics
Summer Semester 2022
Prof. Dr. Anton Bovier,
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Stochastic Processes
Sheet 1

For discussion in the tutorials in the 2nd week of the lecture period

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General information

- Important announcements and up-to-date information can always be found on the IAM homepage.
- Please sign in for exactly ONE of the four exercise class via **ecampus**. The registration will open April 5, 12:00.
- The exercise classes will start in the second week of the lecture period.
- Beginning with sheet 2, you have to hand in your solutions to the problems, in groups of two or three people and electronically via ecampus. More about this in your first tutorial class.
- If you have any questions left regarding the organisation of the lecture, please write an email to florian.kreten@uni-bonn.de.

Exercise 1

Let $\mathbb{P}$ and $\mathbb{Q}$ be probability measures on $(\mathbb{R}; \mathcal{B}(\mathbb{R}))$ that agree on all intervals of the form $(x, y]$, where $-\infty \leq x \leq y < \infty$. Prove that the two probability measures are equal.

Exercise 2

Let $(\Omega, \mathcal{F}, \mu)$ be a σ-finite measure space, λ be the Lebesgue measure on $(\mathbb{R}, \mathcal{B})$ and $f : \Omega \to \mathbb{R}_+$ be a non-negative measurable function. Use Fubini's theorem to prove that

$$\int f d\mu = \int_0^\infty \mu(f \geq t)\, \lambda(dt).$$

Conclude that for all $p \geq 1$:

$$\int f^p d\mu = \int_0^\infty p t^{p-1} \mu(f \geq t)\, \lambda(dt).$$

Exercise 3

Let X, Y be two independent normally distributed random variables with means μ_X, μ_Y and variances $\sigma_X, \sigma_Y > 0$. Show that the sum $Z := X + Y$ is normally distributed with mean $\mu_X + \mu_Z$ and variance $\sigma_X + \sigma_Y$.