

8. Übungsblatt „Mathematik III für Physiker“

Due on (12.12.2011) in exercise class

Class exercise. Find poles, removable and essential singularities of the following functions:

$$f_1(z) = \frac{z^4}{(z^4 - 16)^2}, \quad f_2(z) = \frac{z}{\sin z}, \quad f_3(z) = \frac{1 - \cos z}{\sin z}.$$

1. (Poles and singularities)

[6 Pkt]

Find poles, removable and essential singularities of the following functions:

a) $g_1(z) = \frac{z^2 - \pi^2}{(\sin z)^2}$

b) $g_2(z) = \frac{1}{e^z - 1} - \frac{1}{z}$

c) $g_3(z) = \exp\left(\frac{z}{1-z}\right)$

Hint: consider the sequences $\{z_n = \frac{n-1}{n}, n \in \mathbb{N}\}$ and $\{w_n = \frac{n}{n-1}, n \in \mathbb{N}\}$.

2. (Complex integration)

[6 Pkt]

Consider the function

$$f(z) = \frac{e^{tz}}{z^2(z^2 + 2z + 2)}$$

for $t \in \mathbb{C}$.

a) Find all singularities and residues f .

b) Compute with the help of the residues theorem

$$\int_{|z|=3} f(z) dz.$$

3. (Residues)

[6 Pkt]

Compute

a)

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a)(x^2 + b)}, \quad 0 < a < b.$$

b)

$$\int_0^{2\pi} \frac{d\theta}{\alpha + \beta \sin \theta}, \quad \alpha > \beta > 0$$

*Hint: as in classes, use the formula $\sin z = \frac{e^{iz} - e^{-iz}}{2i}$.***4. (Rouché's Theorem)**

[2 Pkt]

How many zeroes does the following polynom have

$$P(z) = 3z^4 - 7z + 2$$

on $\{z \in \mathbb{C} : 1 < |z| < \frac{3}{2}\}$?