

7. Übungsblatt „Mathematik III für Physiker“

Due on (5.12.2011) in exercise class

Class exercise.

- a) Let $\alpha > 0$. Compute

$$\int_0^\infty \frac{x \sin x}{x^2 + \alpha^2} dx$$

- b) Consider the polynom $P(z) = z^{20} + 5z^8 - 3z^2 + 13z + 1$. How many zeros does P have on the unit disk $U_1(0)$ centered at the origin?

1. (Residues)

[12 Pkt]

Compute the following integrals:

a)

$$\int_0^\infty \frac{x \sin x}{x^2 - \beta^2} dx \quad \text{for } \beta > 0$$

b)

$$\int_0^\infty \frac{1}{t^4 + t^2 + 1} dt$$

c)

$$\int_0^\infty \frac{dx}{(1 + x^2)^3}$$

2. (Residues)

[4 Pkt]

- a) Compute $\int_0^\infty \frac{\sin x}{x} dx$.

- b) Let $n \in \mathbb{N}, n \geq 1$ and $x \in \mathbb{R}$. Show that

$$\sin^{2n+1}(x) = \frac{(-1)^n}{2^{2n+1}} \sum_{k=0}^{2n+1} \binom{2n+1}{k} (-1)^k \sin((2n-2k+1)x)$$

Hint. Use that $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i} = \frac{e^{ix}}{2i} (1 - e^{-2ix})$ and the binomial-formula.

c) Let $m, n \in \mathbb{N}$ and $m < n$. Show with induction on m that

$$\sum_{k=0}^m (-1)^k \binom{n}{k} = (-1)^m \binom{n-1}{m}.$$

d) Use a), b), c) and the residues formula to compute

$$\int_0^\infty \frac{\sin^{2n+1}(x)}{x} dx.$$

3. (Rouché's Theorem)

[4 Pkt]

Consider the polynom $Q(z) = z^{20} + 13z^{17} + 5z^8 - 3z^2 + 1$. How many zeros does Q have on the unit disk $U_1(0)$ centered at the origin?