

# Praktikum Klasse 1

$$1a) \quad a_n = n^{\alpha} \quad \lim_{n \rightarrow \infty} \frac{a_n}{a_{n+1}} = \limsup \left( \frac{n}{n+1} \right)^{\alpha} = \underline{\underline{1}}$$

$$1b) \quad \frac{n!}{n^n} \cdot \frac{(n+1)^{n+1}}{(n+1)!} = \limsup \left( \frac{n+1}{n} \right)^n = \underline{\underline{e}}$$

$$1c) \quad \begin{cases} \infty & |\alpha| < 1 \\ 1 & |\alpha| = 1 \\ 0 & |\alpha| > 1 \end{cases}$$

$$1d) (*) = \sum_m (-1)^m 2^m z^{m+2} = z^2 \sum_m (-1)^m 2^m (z^2)^m. \quad \text{Betrachte } \sum (-1)^m 2^m w^m, \quad w = z^2$$

Konvergenzradius  $\limsup \left| \frac{(-1)^m 2^m}{(-1)^{m+1} 2^{m+1}} \right| = \frac{1}{2}. \quad \text{Also Konvergenzradius von } (*) = \frac{n}{2}.$

$$2a) \quad f(z) = x^2 + iy^2$$

$$\begin{cases} u_x = 2x \\ v_y = 2y \end{cases}$$

$$2b) \quad f(z) = (x+iy)x = x^2 + iyx$$

$$\begin{cases} u_x = 2x \\ v_y = x \end{cases}$$

$$\begin{cases} u_y = 0 \\ v_x = 0 \end{cases}$$

$\Rightarrow$   $f$  diff'bar nur in  $(0,0)$

$$3a) \quad \int_0^2 x^2 dx = \frac{x^3}{3} \Big|_0^2 = \underline{\underline{8}}$$

denn  $\int_{\gamma}^{1+i} t^{n+i} dt \equiv 0$ !

$$3b) \quad \int_{\sigma_1}^2 z^k dz = \int_{\sigma_1}^2 z^k dt + \int_{\sigma_1}^2 z^k dt = \int_0^1 (1-i)t (n+i) dt + \int_0^2 ((n-i)(n-t) + 2t) (1-i) dt = (1-i) = \underline{\underline{2(n-i)}}$$

$$4) \quad \text{es gilt} \quad f(a) = \frac{1}{2\pi i} \int_{\gamma} \frac{dt}{(t-a)(t-1/a)}$$

Fall,  $|a| < 1$   $\Re \frac{1}{a} > 1$

$$= \frac{1}{2\pi i} \int_{\gamma} \frac{(t-1/a)^{-1}}{(t-a)} dt = (a - \frac{1}{a})^{-1} = \frac{a}{a^2 - 1} \quad \neq$$

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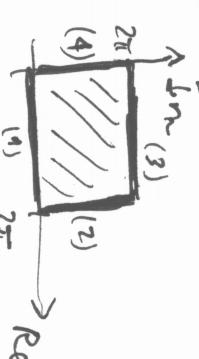
$$5) \quad \sup \{ | \sin z | : \operatorname{Re}(z) \in [0, \infty], \operatorname{Im}(z) \in [0, 2\pi] \} = \underline{\underline{1}}$$

$$\sup(1) | \sin z | = \sup \{ \sin \theta : \theta \in [0, 2\pi] \} = \underline{\underline{1}}$$

$$\sup(2) | \sin z | = \frac{e^{2\pi} - e^{-2\pi}}{2} \quad \sup(3) = \frac{1}{2} \sqrt{\lambda + 2\lambda e^{4\pi} + e^{8\pi}}$$

$$\sup(3) = \frac{e^{2\pi} - e^{-2\pi}}{2}$$

max auf dem Rand!



$$= \frac{1}{2\pi i} \int_{\gamma} \frac{(t-1/a)^{-1}}{(t-a)} dt = (a - \frac{1}{a})^{-1} = \frac{a}{a^2 - 1} \quad \neq$$

$$\sup(3) = \frac{e^{2\pi} - e^{-2\pi}}{2}$$

$$6a) f(z) = \frac{1}{e^z - e}$$

$e^z = e \Rightarrow z_u = n + 2\pi i, u \in \mathbb{R} \rightarrow z_u$  sind Pole (1. Art Ordnung)

$$6b) f(z) = \frac{1 - \cos z}{\sin^2 z}$$

$1 - \cos z = 0 \Leftrightarrow z_u = 2\pi u, u \in \mathbb{Z}$  treten da  $\frac{d^2}{dz^2}(1 - \cos z)|_{z=2\pi u} = 1 \neq 0$   
 $\sin^2 z = 0 \Leftrightarrow z_u = k\pi, u \in \mathbb{Z}$  treten da  $\frac{d}{dz}\sin^2 z|_{z=k\pi} = (-1)^u \neq 0$

Aber  $z_u = \begin{cases} \text{Pol 2. Ordnung} & \text{u gerade} \\ \text{Nullst.} & \text{u ungerade} \end{cases}$

$$7) \int_0^{2\pi} \frac{\cos^2(x)}{x + \cos(x)} dx = \int_0^{2\pi} \frac{(e^{ix} + e^{-ix})^2}{4(x + e^{ix} + e^{-ix})} dx = \int_0^{2\pi} \frac{(x^2 + 1)^2}{x^2(2x + 2 + 1/x)} \frac{dx}{x} =$$

$$= \pi \int_{-\pi i}^{\pi i} \frac{(x^2 + 1)^2}{x^2(x^2 + 2x + 1)} dx = \left( \text{Bestimmung der NS, etc...} \right) = \pi \left( \frac{\sqrt{x^2 + 1}}{x} - 1 \right)$$

$$8a) \int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)^2} dx = ? \quad \xrightarrow{\text{Drehung}}$$

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)^2} dx = 2\pi i \operatorname{Res} \left( \frac{x^2}{(x+i)^2(x-i)^2}, z=i \right) = \dots = \frac{\pi}{a^2}$$

$$8b) \int_{-\infty}^{\infty} \frac{x^2}{(x^2 + a^2)(x^2 + b^2)} dx = \frac{\pi}{a+b} // \quad (\text{ähnlich wie oben})$$

$$9a) \frac{1}{z^2 + 1} = \frac{1}{(z-i)(z+i)} = \frac{1}{(z-i)+2i} = \frac{1}{z-i} \cdot \frac{1}{1+\frac{2i}{z-i}} = \frac{1}{2i} \cdot \frac{1}{(z-i)} \sum_{n=0}^{\infty} (-1)^n \left(\frac{2i}{z-i}\right)^n = \frac{1}{4} \sum_{n=1}^{\infty} \left(\frac{i}{2}\right)^n (z-i)^{-n}$$

$$9b) \text{ähnlich} = \frac{1}{4} \sum_{n=1}^{\infty} \left(-\frac{i}{2}\right)^n (z+i)^{-n} //$$

$$10) \left\{ -y''(x) = \lambda y(x) \quad 0 \leq x \leq L \right.$$

$$\left. y(0) = 0, \quad y(L) \sin(\beta) = y'(L) \cos(\beta) \right.$$

Allgemeine Lösung:  $\underline{\lambda > 0}$ ,  $y(x) = A \cos(\sqrt{\lambda} x) + B \sin(\sqrt{\lambda} x)$

$$\underline{\lambda = 0}, \quad y(x) = Cx + D$$

$$\underline{\lambda < 0}, \quad y(x) = E e^{\sqrt{-\lambda} x} + F e^{-\sqrt{-\lambda} x}$$

$$\underline{\text{Fall } \lambda > 0} \quad y(0) = 0 \rightarrow A = 0, \quad y(L) \cos \beta = y'(L) \sin \beta$$

$$B \sin(\sqrt{\lambda} L) \cos \beta = B \sqrt{\lambda} \cos \sqrt{\lambda} L \sin \beta \quad , \quad B \neq 0 .$$

$$\underline{\text{Fall } \beta = \frac{\pi}{2}} \quad \text{keine Lösung, } \beta \neq \frac{\pi}{2} \quad \tan(\beta) = \frac{\tan(\sqrt{\lambda} L)}{\sqrt{\lambda}}$$

$$\underline{\text{Fall } \lambda = 0} \quad \text{Allgemein: } \beta = \frac{\pi}{2} \quad \text{keine Lösung, } \beta \neq \frac{\pi}{2} \Rightarrow \tan(\beta) = L$$

$$\underline{\text{Fall } \lambda < 0} \quad \beta = \frac{\pi}{2} \quad \text{keine Lösung} \quad \left| \begin{array}{l} \beta \neq \frac{\pi}{2} \\ -\frac{\pi}{2} < \beta < \frac{\pi}{2} \end{array} \right. \quad \rightarrow \quad \tan(\beta) = \frac{1}{\sqrt{-\lambda}} \tan(\sqrt{-\lambda} L) \\ (\beta) > \frac{\pi}{2} \quad \text{es nur } \tan(\beta) > 0$$

$$10b) \quad \lambda_{\min} < 0 \rightarrow \beta \in (0, \frac{\pi}{2}) \quad \text{und } \tan(\beta) < L$$

10c) Ans. Monotonie Untersuchung nach dass kleinster EW für  $\underline{\beta \rightarrow 0}$