**Institut für Angewandte Mathematik**
**Markov Processes**

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# Exercise sheet 9

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**Abgabe:** 26.06.2012

## Exercise 1 **(7 Points)**

Let $0 < a < b$ and consider the operator

$$\mathcal{D}(G) = C^2([0,1]), \qquad Gf(x) = x(1-x)f''(x) + (a - bx)f'(x)\ .$$

Use duality to show uniqueness for the martingale problem for $G$.

**Hint**: Take $\{0, 1, 2, \ldots\}$ as a state space for the dual problem and $f(x, y) = x^y$.

## Exercise 2 **(7 Points)**

Let $b_i$ and $\sigma_{ij}$, $1 \leq i, j \leq d$, be progressively measurable function from $[0, \infty) \times C[0, \infty)^d$ into $\mathbb{R}$. Such that

$$\|b(t, y)\|^2 + \|\sigma(t, y)\|^2 \leq C\left(1 + \max_{0 \leq s \leq t} \|y(s)\|^2\right), \quad 0 \leq t < \infty, y \in C[0, \infty)^d,$$

where $C$ is a positive constant. Furthermore, let the stochastic integral equation

$$X_t = X_0 + \int_0^t b(x, X_s)ds + \int_0^t \sigma(s, X_s)dB_s, t \in \mathbb{R}_+,$$

have a weak solution, with $E[\|X_0\|^{2m}] < \infty$ for some $m \geq 1$. Prove that for every finite time $T > 0$, we have

$$E\left(\max_{0 \leq s \leq t} \|X_s\|^{2m}\right) \leq Ke^{Kt}\left(1 + E[\|X_0\|^{2m}]\right), \quad 0 \leq t \leq T,$$

$$E\left(\|X_t - X_s\|^{2m}\right) \leq K\left(1 + E[\|X_0\|^{2m}]\right)(t - s)^m, \quad 0 \leq s < t \leq T,$$

where $K$ is a positive constant depending only on $m, T, C$ and $d$.

## Exercise 3 **(6 Points)**

Prove that Theorem 4.15 is still valid , if the condition that the coefficients $b_i$ and $\sigma_{ij}$, $1 \leq i, j \leq d$, are bounded, is replaced by

$$\|b(y)\|^2 + \|\sigma(y)\|^2 \leq C(1 + \|y\|^2),$$

and the initial distribution $\mu$ satisfies

$$\int_{\mathbb{R}^d} \|x\|^{2m}\mu(dx) < \infty, \text{ for some } m > 1.$$

Gesamt: 20 Punkte