Institute for Applied Mathematics
Summer term 2014
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"Stochastic Processes", Problem Sheet 4.

Hand in solutions before Wednesday 7.5., 2 pm.
(post-box opposite to maths library)

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1. (Poissonian bears)
At time $t = 0$ there aren't any bears in a village. Brown bears und grizzly bears arrive as independent Poisson processes B and G with intensities β and γ.

a) Show that with probability $\beta/(\beta + \gamma)$ the bear arriving first is a brown bear.

b) Determine the probability that between two succeeding brown bears exactly r grizzly bears arrive in the village.

c) Compute the conditional expectation of the arrival time of the first bear given that $B_1 = 1$.

2. (Poissonian forest)
Let N be a Poisson process on $\mathbb{R}^2$ with homogeneous intensity measure $\lambda\, dx$, $\lambda > 0$, and let $R_{(1)} < R_{(2)} < \dots$ be the increasingly ordered distances from the points of the Poisson process to the origin.

a) Show that $R^2_{(1)}, R^2_{(2)}, \dots$ are the points of a Poisson process on $[0, \infty)$ with intensity $\pi\lambda$.

b) Prove directly or with the help of a), that the density of $R_{(k)}$ has the following form:

$$f(r) = \frac{2\pi\lambda r(\lambda\pi r^2)^{k-1} e^{-\lambda\pi r^2}}{(k-1)!}, \quad r > 0.$$

3. (Limit theorems for conditional expectations) State and prove:

a) A monotone convergence theorem for conditional expectations.

b) Fatou's lemma for conditional expectations.

c) A dominated convergence theorem (Lebesgue's theorem) for conditional expectations.

4. (Simulation of Poisson Processes)
Generate simulations (e.g. with Mathematica) of

a) Poisson Point Processes on $[0, 1]^2$ with intensity measures $\lambda\, dx$, $\lambda \in \mathbb{R}_+$,

b) Poisson Processes on $[0, t]$ with intensities $\lambda > 0$.

Visualize your results, including variations of the parameters.