Institut für angewandte Mathematik
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„Stochastic Processes", Problem Sheet 1.

Please hand in your solutions before 3 pm on Tuesday, April 17, into the marked post boxes opposite to the maths library.

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1. (Conditional Expectations).
Let $X, Y : \Omega \to [0, \infty)$ be independent identically distributed (iid) discrete random variables with expectation m.

a) Find the mistake in the following reasoning:

$$E[X \,|\, X+Y=z] \;=\; E[X \,|\, X=z-Y] \;=\; E[z-Y] \;=\; z-m.$$

b) Show that

$$E[X \,|\, X+Y] \;=\; \frac{1}{2}(X+Y).$$

c) Can one proof in a similar way that

$$E[X \,|\, X \cdot Y] \;=\; (X \cdot Y)^{1/2} \;?$$

2. (Error Detection). A factory is producing notebooks that are defect with probability p. A test identifies defect notebooks with probability $1-\varepsilon$ and non-defect notebooks with probability 1.

a) Show that the probability that a notebook which passed the test is nevertheless defect, is $\varepsilon p/(1-p+\varepsilon p)$.

b) The factory produces n notebooks a day. Let X denote the number of defect notebooks, and let Y be the number of notebooks identified as defect. Under suitable independence assumptions show that

$$E[X \,|\, Y] \;=\; Y + (n-Y) \cdot \frac{\varepsilon p}{1-p+\varepsilon p} \;=\; \frac{\varepsilon p n + (1-p)Y}{1-p+\varepsilon p}.$$

3. (Transformations of exponential random variables).
Let T and R be independent exponentially distributed random variables with parameters λ and μ respectively. Determine

a) the conditional distribution of T given $T+R$,

b) the distribution of T/R.

4. (Properties of conditional expectations).

Let $Y : \Omega \to S$ be a discrete random variable, and let $X : \Omega \to \mathbb{R}$ be an integrable real-valued random variable, both defined on a common probability space $(\Omega, \mathcal{A}, P)$. Prove that:

a) The map $X \to E[X|Y]$ is almost surely linear and monotone.

b) If $X = \tilde{X}$ almost surely, then also $E[X|Y] = E[\tilde{X}|Y]$ almost surely.

c) For any $f : S \to \mathbb{R}$ such that $f(Y) \cdot X \in \mathcal{L}^1$,

$$E[f(Y) \cdot X|Y] \;=\; f(Y) \cdot E[X|Y] \qquad P\text{-a.s.}$$

Which result do we obtain if X and Y are independent ?

5. (Literature check).

Go to the maths library and have a look at textbooks on stochastic processes, e.g. those recommended below. Reference copies of these books will be placed behind the counter at the library entrance.

Recommended textbooks on Stochastic Processes:

- Breiman: Probability.
- Durrett: Probability: Theory and Examples.
- Grimmett/Stirzaker: Probability and Random Processes.
- Kersting/Wakolbinger: Stochastische Prozesse
- Klenke: Wahrscheinlichkeitstheorie.
- Koralov/Sinai: Theory of Probability and Random Processes.
- Norris: Markov Chains.
- Williams: Probability with Martingales.