

The course will focus on the analysis of diffusion processes. A prototypical recurrent example is the Langevin process (X_t, V_t) that solves an SDE of the form

$$\begin{aligned}dX_t &= V_t dt \\dV_t &= -b(X_t) dt - \gamma V_t dt + (2\gamma)^{1/2} dB_t\end{aligned}$$

with a Brownian motion (B_t) .

Course overview

- Transformations and weak solutions of stochastic differential equations
- Extensions of Itô calculus, stochastic flows, stability of SDE
- Couplings and convergence of diffusion processes
- Functional inequalities and relaxation
- Numerical methods for SDE and (Markov Chain) Monte Carlo

Recommended textbooks

The following textbooks cover all a broad range of topics in stochastic analysis. Nevertheless, there are substantial differences in style and content between them.

- Rogers, Williams : *Diffusions, Markov processes and martingales, Vol. 2: Ito calculus*, Cambridge UP.
- Bass : *Stochastic Processes*, Springer.
- Protter : *Stochastic integration and differential equations*, Springer.
- Revuz, Yor : *Continuous martingales and Brownian motion*, Springer.
- Le Gall: *Brownian motion, martingales, and stochastic calculus*, Springer.
- Karatzas, Shreve : *Brownian motion and stochastic calculus*, Springer.
- Ikeda, Watanabe: *SDE and diffusion processes*, North Holland.
- Jacod/Shiryayev: *Limit Theorems for Stochastic Processes*, Springer.

Special topics:

- Pavliotis: *Stochastic Processes*, Springer. (Langevin dynamics)
- Bakry, Gentil, Ledoux: *Analysis and geometry of Markov diffusion operators*
- Da Prato: *Introduction to Stochastic Analysis and Malliavin Calculus*, SNS Pisa. (Infinite dimensional analysis, Malliavin calculus)
- Friedman: *Stochastic Differential Equations and Applications*, Dover.
- Liptser/Shiryayev: *Statistics of Random Processes I and II*, 2 nd Ed., Springer.
- Shreve: *Stochastic Calculus for Finance II*, Springer.