S4F6 Graduate Seminar on Stochastic Processes WS 2025/26

Malliavin calculus and Mean-field limits

Andreas Eberle Fridays 14-16, Room N0.007

Preliminary meeting: Tuesday 15.7., 9.30, KHS We.10

In this seminar we consider two different topics in stochastic analysis.

The first topic is "Malliavin calculus". Here one introduces analytic structures (gradient, divergence, Sobolev spaces) on the Wiener space. These are closely connected to variations of stochastic differential equations, i.e., differentiation of the solutions with respect to parameters. Malliavin calculus provides a probabilistic proof of Hörmander's theorem on the existence of smooth densities for solutions to hypoelliptic PDE, and it has applications to Monte Carlo methods in finance.

In stochastic mean-field dynamics, there is a number N of components/particles which all interact with each other. The dynamics is described for example by a system of coupled stochastic differential equations. Under certain assumptions, a „propagation of chaos" property holds: If the particles are independent at time zero then in the limit as N tends to infinity, asymptotic independence holds at any fixed time t. In this case, the dynamics of a single particle can be asymptotically described by a stochastic differential equation where the coefficients depend on the law of the process, and, correspondingly, the asymptotic laws of the processes at time t satisfy a nonlinear partial differential equation. Applications include numerical methods for filtering, models in physics and biology, mean-field games, and neural networks.

References:

a. Malliavin calculus

- M. Hairer, *Introduction to stochastic analysis and Malliavin calculus,* Lecture Notes, https://www.hairer.org/notes/Malliavin.pdf
- D. Nualart, *The Malliavin calculus and related topics*, 2nd ed., Springer (2005)
- G. Da Prato: *Introduction to stochastic analysis and Malliavin calculus*.

b. Mean-field limits

- A.S. Sznitman: *Topics in propagation of chaos*, École d'été St.Flour 1989, Springer LNM 1464

Prerequisites: „Introduction to Stochastic Analysis".