S4F2 Graduate Seminar on Stochastic Analysis

Summer Semester 2026

Stochastic localization

Andreas Eberle Thursdays 14-16, Room 0.011

Preliminary meeting: Thursday 29.1., 13.30, GHS We.10

Stochastic localization processes are probability measure valued martingales $(\mu_t)_{t\in[0,\infty)}$ that interpolate between a deterministic probability measure μ at time $t=0$ and a Dirac measure at a random sample drawn from μ in the limit as $t \to \infty$. The concept has been introduced by Eldan in 2013. Since then, connections to various areas of pure and applied mathematics have been found, see [1].

In particular, denoising diffusion models (score-based generative models) [6] rely on simulating a time reversed stochastic differential equation related to a stochastic localization process. Significant recent progress on the KLS conjecture, a famous conjecture on log-concave probability measures, is due to stochastic localization [2,3]. More generally, stochastic localization provides a powerful tool to study functional inequalities and convergence to equilibrium for Markov chains [4]. This approach turns out to be closely connected to the renormalization group approach to logarithmic Sobolev inequalities developed in [5].

The seminar will give and introduction to stochastic localization and its various applications.

References:

1. Shi, Tian, Zhang: Perspectives on Stochastic Localization, arXiv 2025
2. Lee, Vempala: Eldan's stochastic localization and the KLS conjecture: Isoperimetry, concentration and mixing, Annals of Mathematics (2024)
3. Chen: An almost constant lower bound for the isoperimetric constant in the KLS conjecture, GFA 2021
4. Chen, Eldan: Localization Schemes: A Framework for Proving Mixing Bounds for Markov Chains, Duke Math. J. (2025)
5. Bauerschmidt, Bodineau, Dagallier: Stochastic dynamics and the Polchinski equation: an introduction, Prob. Surveys (2024)
6. Montanari: Sampling, diffusions and stochastic localization, arXiv 2023
7. Lai, Song, Kim, Mitsufuji, Ermon: The principles of diffusion models, arXiv 2025

Prerequisites: „Introduction to Stochastic Analysis".