Institut für Angewandte Mathematik
Winter semester 2025/26
Andreas Eberle, Francis Lörler

"Markov Processes", Problem Sheet 12

Please hand in your solutions by Friday, January 23, 13.00.

---

1. (Interacting particle systems on finite graphs).

a) Show that for the voter model on a finite graph (V, E), the processes

$$N_i(\eta_t) \;=\; \sum_{x\in V} 1_i(\eta_t(x))$$

are martingales. Conclude that for all i and all initial configurations ξ,

$$\mathbb{P}_\xi\left[\eta_t \equiv i \text{ eventually}\right] \;=\; \frac{N_i(\xi)}{|V|}.$$

b) Prove for the contact process on a finite graph that for all $\xi \in \{0,1\}^V$,

$$\mathbb{P}_\xi\left[\eta_t \equiv 0 \text{ eventually}\right] \;=\; 1.$$

2. (Independent particle process). Suppose that $\left(X_t^k\right)_{t\geq 0}$ $(k \in \mathbb{N})$ are independent continuous time simple random walks on $\mathbb{Z}^d$, and let

$$\eta_t(x) \;=\; \left|\{k\in\mathbb{N} : X_t^k = x\}\right| .$$

a) Assuming that $\eta_t(x)$ is finite for all $x \in \mathbb{Z}^d$ and $t \geq 0$, show that $(\eta_t)_{t\geq 0}$ is a Markov process with state space $\{0,1,2,\ldots\}^{\mathbb{Z}^d}$.

b) On which Banach space does the process generate a strongly continuous contraction semigroup?

c) Identify the generator on cylinder functions.