StochasticIntegrals

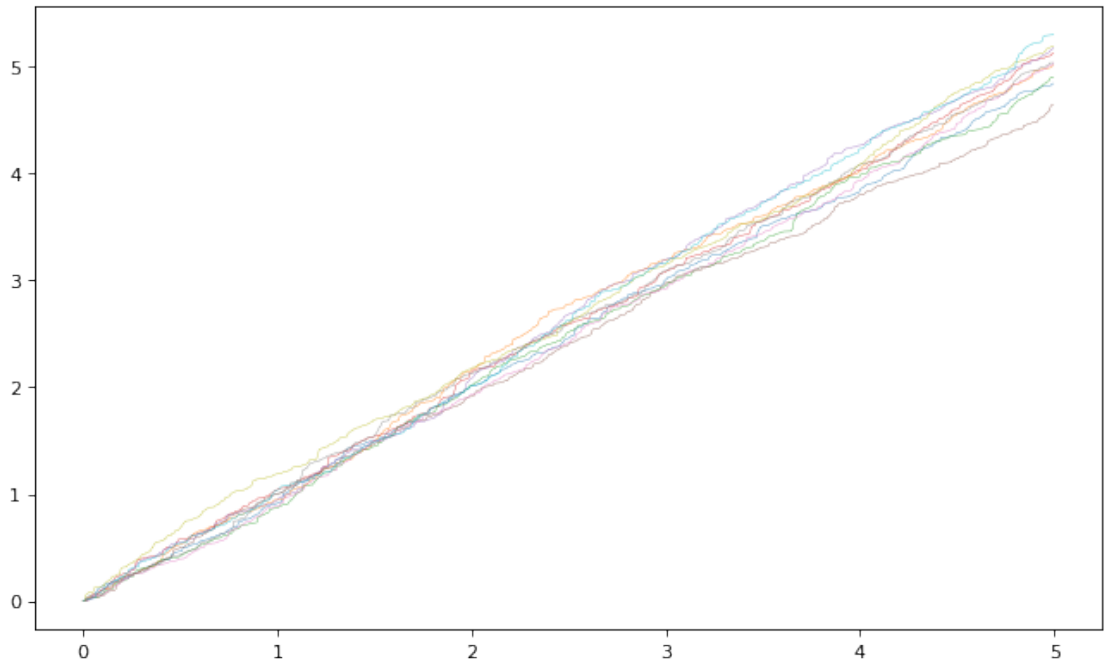
November 9, 2018

1 Simulation of Ito vs. backward Ito integrals

We simulate the Ito and backward Ito integrals of a Brownian motion w.r.t. itself.

In [17]:
```
import numpy as np
# makes numpy routines and data types available as np.[name of routine or data type]

import matplotlib.pyplot as plt
# makes plotting command available as plt.[name of command]

tmax = 5.
# simulation of process from time 0 to tmax
stepslist = [1000,10000]
# produce simulations with step numbers chosen from steplist

for steps in stepslist:
    h = tmax/steps
    # stepsize for time discretization
    std = np.sqrt(h)
    # standard deviation for the distribution of each step

    k = 10
    # number of samples that will be simulated

    noise = np.random.randn(steps,k)*std
    # create a steps times k dimensional matrix of normal random numbers
    # with variance h


    bm = np.zeros((steps,k))
    itoint = np.zeros((steps,k))
    bwint = np.zeros((steps,k))
    # initialize the arrays for Brownian motion, Ito and backward Ito integral

    for n in range(steps-1):
        bm[n+1] = bm[n]+noise[n]
        itoint[n+1] = itoint[n]+bm[n]*noise[n]
```

```
    bwint[n+1] = bwint[n]+bm[n+1]*noise[n]

t = np.arange(0,steps,1)*h
# creates vector of time points

plt.figure(figsize=(10,6), dpi=80)
# sets size of plot
plt.plot(t,itoint,linewidth=0.3)
plt.plot(t,bwint,linewidth=0.3)
# produces plots of Ito and backward Ito integral versus t
plt.show()
# output of plot

plt.figure(figsize=(10,6), dpi=80)
# sets size of plot
plt.plot(t,bwint-itoint,linewidth=0.3)
# produces a plot of the difference between backward Ito and Ito integral
plt.show()
# output of plot
```


0
1
2
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4
5
0
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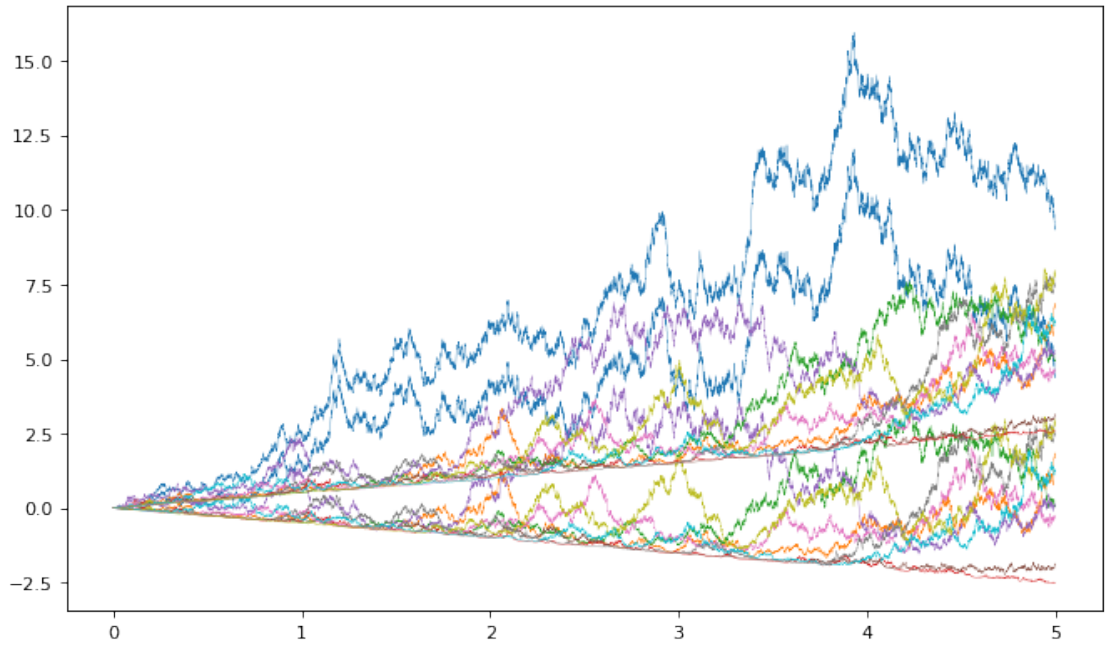
5


15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
0
1
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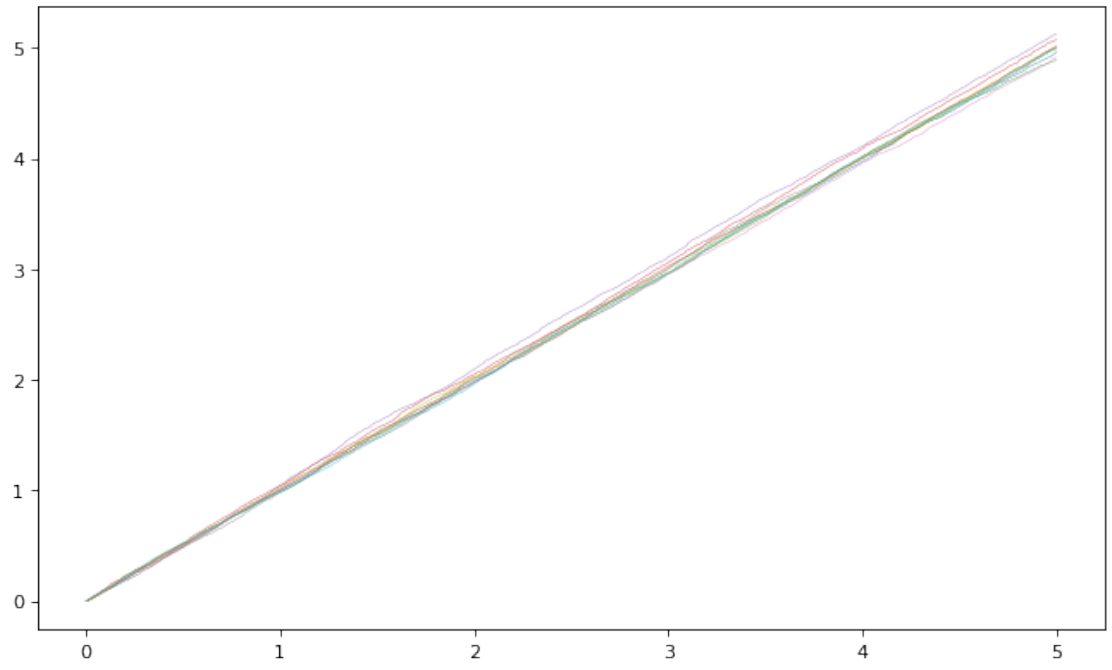
3
4
5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