Introduction to Stochastic Analysis **Andreas Eberle**

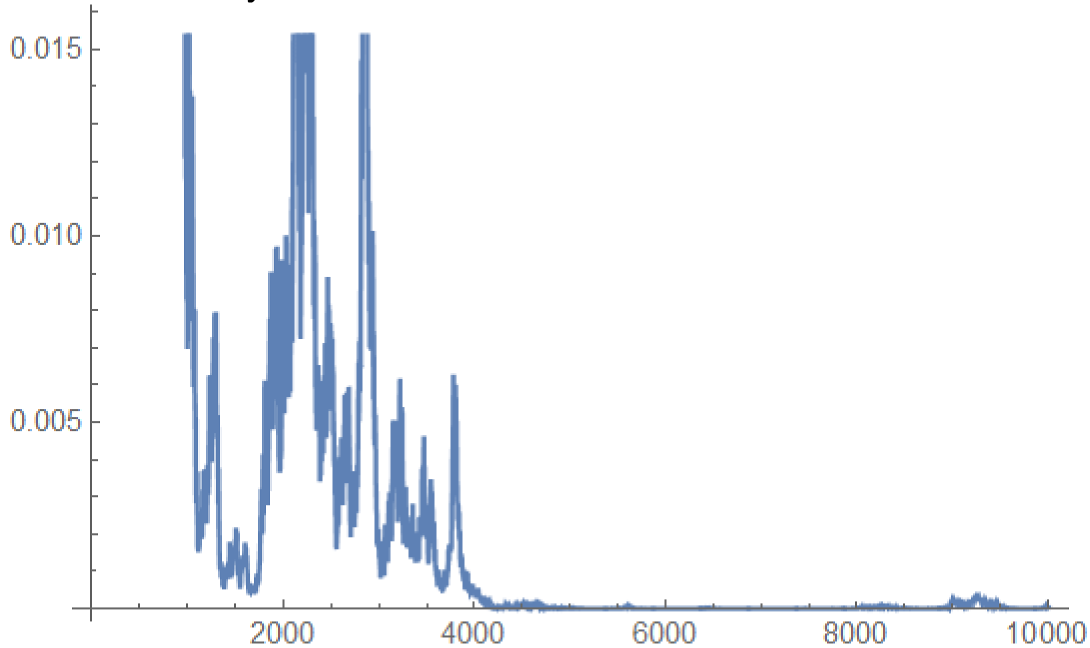
In stochastic analysis, more general stochastic processes in continuous time are constructed from Brownian motion. Stochastic differential calculus (Itō calculus) enables us to compute corresponding expectation values in an elegant way, and to set up connections to differential equations. Stochastic Analysis is important for many application areas (including mathematical finance, but also natural sciences and engineering). It also has fundamental connections to many other mathematical disciplines, and it is the basis for most probability courses in the Master programme.

Contents and prerequisites:

I plan to cover Chapters 3-9 of the lecture notes on my webpage

http://wt.iam.uni-bonn.de/eberle/skripten/

Chapter 1 on Brownian motion will not be discussed during this course, since this material is part of the „Stochastic Processes" course. Knowledge of the basics about Brownian motion will be assumed from week 5 onwards. During the first 4 weeks of the course, an introduction to martingales in continous time will be given. Measure theoretic probability and conditional expectations w.r.t. σ-algebras are required as a prerequisite, see for instance Appendix A in the lecture notes or Chapter 9 in „Probability with martingales" by D. Williams. In the beginning, we will briefly review the material on martingales in discrete time in Sections 2.1-2.3 of the lecture notes which has already been discussed in the Stochastic Processes course.



The figure shows a typical sample path of geometric Brownian motion. The process is an exponential martingale. In particular, the expectations are constant in time, but nevertheless the sample paths converge to 0 with probability one.