Institute for Applied Mathematics, Bonn University

Oberseminar Stochastik

Thursday, 09 December 2021, 16:30
online

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Langevin Processes in Bounded-in-Position Domains—Application to Quasi-Stationary Distributions

Quasi-stationary distributions can be seen as the first eigenvector associated with the generator of the stochastic differential equation at hand, on a domain with Dirichlet boundary conditions (which corresponds to absorbing boundary conditions at the level of the underlying stochastic processes). Many results on the quasi-stationary distribution hold for non degenerate stochastic dynamics, whose associated generator is elliptic. The case of degenerate dynamics is less clear. In this work, together with T. Lelièvre and J. Reygner (Ecole des Ponts, France) we generalize well-known results on the probabilistic representation of solutions to parabolic equations on bounded domains to the so-called kinetic Fokker Planck equation on bounded domains in positions, with absorbing boundary conditions. Furthermore, a Harnack inequality, as well as a maximum principle, is provided for solutions to this kinetic Fokker-Planck equation, as well as the existence of a smooth transition density for the associated absorbed Langevin dynamics. The continuity of this transition density at the boundary is studied as well as the compactness, in various functional spaces, of the associated semigroup. Following these results we obtain the existence of a unique quasi-stationary distribution as well as a convergence result of the Langevin process conditioned on non-exiting the domain towards its quasi-stationary distribution. Using estimates obtained in the particular one-dimensional case, this result can also be sharpened to a uniform convergence result with respect to its initial distribution. This work is a cornerstone to prove the consistency of some algorithms used to simulate metastable trajectories of the Langevin dynamics, for example the Parallel Replica algorithm.