A Multiscale Analysis of Traveling Waves in Stochastic Neural Fields

Neural field equations are used to describe the spatio-temporal dynamics of the activity in synaptically coupled populations of neurons in the continuum limit. They exhibit traveling wave solutions, modeling the propagation of activity. We analyze the behavior of these solutions under the influence of noise. By separating two scales - fluctuations in the wave profile, and shifts in the phase of the wave - we obtain a simplified description of the dynamics. We prove that the nonlocal linear operator associated with the problem satisfies a spectral gap inequality. This will allow us to express the (stochastic) stability of the traveling wave. We find that, to first order of the noise strength, the phase shift is roughly diffusive and the fluctuations in the wave profile are given by a stationary Ornstein-Uhlenbeck process.