Probability and asymptotic analysis in strongly coupled systems

Monday, January 11

8:30 - 9:00	**Registration**
9:00 - 9:10	**Welcome**
9:10 - 10:00	**Lecture 1, part a: Figalli** A transportation approach to random matrices
10:00- 10:30	**Coffee Break**
10:30- 11:30	**Lecture 1, part b: Figalli** A transportation approach to random matrices
11:40- 12:30	**M. Shcherbina** Supersymmetry approach to the analysis of the Ginibre ensemble
12:30- 14:30	**Lunch Break**
14:30- 15:20	**Serfaty** LDP and CLT for fluctuations of Log and Riesz Gases
15:20- 16:10	**Götze** Convergence rate of expected and random spectra in 4+ moment Wigner ensembles
16:10- 16:40	**Coffee Break**
16:40- 17:30	**Mirlin** Anderson transitions, multifractality, and electron-electron interaction

Tuesday, January 12

9:00 - 9:50	**Lecture 2, part a: Knowles** Local eigenvalue statistics for random regular graphs
9:50 - 10:30	**Group Picture + Coffee Break**
10:30- 11:30	**Lecture 2, part b: Knowles** Local eigenvalue statistics for random regular graphs
11:40- 12:30	**Bao** Local law of addition of random matrices
12:30- 15:00	**Lunch Break**
15:00- 15:50	**Kieburg** What is the relation between eigenvalues & singular values ?
15:50- 17:00	**Coffee Break**
16:00- 17:00	**Poster session**
17:00- 17:50	**Akemann** Products of random matrices: dropping the independence
18:00- open	**Reception**

Wednesday, January 13

9:00 - 9:50	**Lecture 3, part a: Zirnbauer** Introduction to superbosonization
9:50 - 10:20	**Coffee Break**
10:20- 11:20	**Lecture 3, part b: Zirnbauer** Introduction to superbosonization
11:30- 12:20	**Open problem discussion**
Afternoon	**Free time (resp. Excursion)**

Thursday, January 14

9:00 - 9:50 **Lecture 4, part a: Kozłowski**
Asymptotic expansion of the sinh-model with varying interactions

9:50 - 10:20 **Coffee Break**

10:20 - 11:20 **Lecture 4, part b: Kozłowski**
Asymptotic expansion of the sinh-model with varying interactions

11:30 - 12:20 **Kitanine**
Form factor approach to the asymptotic analysis of correlation functions in massless quantum integrable models

12:20 - 14:30 **Lunch Break**

14:30 - 15:20 **Gorin**
Largest eigenvalues in random matrix beta-ensembles: structures of the limit

15:20 - 16:10 **Péché**
Universal versus non-universal features in random matrix theory via deformed ensembles

16:10 - 16:40 **Coffee Break**

16:40 - 17:30 **Fyodorov**
On the height and position of maxima of GUE characteristic polynomials

Friday, January 15

9:00 - 9:50 **Chafaï**
At the edge of interacting particle systems

9:50 - 10:40 **Moll**
Random Partitions and the Quantum Benjamin-Ono Hierarchy

10:40 - 11:10 **Coffee Break**

11:10 - 12:00 **Soshnikov**
Spectral properties of products of large random matrices with independent entries

12:10 - 13:00 **Sodin**
Semi-classical analysis of non-self-adjoint Kac operators

Talks - Abstracts

Alessio Figalli (Univ. Texas, Austin) - A transportation approach to random matrices (2h)

Optimal transport theory is an efficient tool to construct change of variables between probability densities. However, when it comes to the regularity of these maps, one cannot hope to obtain estimates that are uniform with respect to the dimension except in some very special cases (for instance, between uniformly log-concave densities). In random matrix theory the densities involved (modeling the distribution of the eigenvalues) are pretty singular, so it seems hopeless to apply optimal transport theory in this context. However, ideas coming from optimal transport can still be used to construct approximate transport maps (i.e., maps which send a density onto another up to a small error) which enjoy regularity estimates that are uniform in the dimension. Such maps can then be used to show universality results for the distribution of eigenvalues in random matrices. The aim of these lectures is to give an overview of these results.

Mariya Shcherbina (ILTPE Kharkiv) - Supersymmetry approach to the analysis of the Ginibre ensemble

We consider an application of the Grassmann integration method to problems of local and global regimes of the Genibre ensemble. We discuss the proof of the circular low by this method and a possibility to prove Central Limit Theorem for linear eigenvalue statistics for the ensemble.

Sylvia Serfaty (Paris Jussieu) - LDP and CLT for fluctuations of Log and Riesz Gases

We present a Large Deviation Principle for large systems of particles with logarithmic interactions in 1D and 2D, or more general inverse power (= Riesz) interactions, which can be called Log gases and Riesz gases. The former, also called beta-ensembles, are of particular interest due to their connection to Random Matrix Theory. The LDP lies at next to leading order and is expressed in terms of the microscopic point processes or "empirical fields". In the case of the 2D logarithmic interaction, we also present a Central Limit Theorem for the fluctuations from the macroscopic distribution. All the results are valid for arbitrary values of the inverse temperature, and fairly general confining potentials. This is joint work with Thomas Leblé.

Friedrich Götze (Bielefeld Univ.) - Convergence rate of expected and random spectra in 4+ moment Wigner ensembles

The rate of convergence of the spectrum of Wigner matrices with 4+ moments to the semicircular law is investigated. An optimal rate is shown for the convergence of the expected spectrum as well as (local) rates for random spectra. Applications to the convergence to the Marcenko-Pastur laws are discussed. This is joint work with Alexey Naumov and Alexander Tikhomirov.

Alexander Mirlin (Karlsruhe IT) - Anderson transitions, multifractality, and electron-electron interaction

I will review the physics of Anderson localization transitions, with a particular focus on multifractality of critical eigenfunctions. In the second part of the talk I will discuss effects of electron-electron interaction at Anderson transitions.

Antti Knowles (ETH Zürich) - Local eigenvalue statistics for random regular graphs

I review recent results on local eigenvalue statistics for random regular graphs, whose degrees are subject to mild growth assumptions. In the first part of the lectures, I state and sketch the proof of a local law, which provides control of the Green function of the adjacency matrix down to the optimal spectral scale. In the second part, I discuss applications to the distribution of eigenvectors and eigenvalues: a local form of quantum unique ergodicity and GOE local spectral statistics. An important tool is a Markovian switching dynamics that is a discrete approximation of Dyson Brownian motion. Joint work with R. Bauerschmidt, J. Huang, and H.-T. Yau.

Zhigang Bao (IST Vienna) - Local law of addition of random matrices

The eigenvalue distribution of the sum of two Hermitian matrices, when one of them is conjugated by a Haar distributed unitary matrix, is given by the free additive convolution of their spectral distributions. In this talk, I will show that this also holds locally in the bulk of the spectrum, down to the optimal scales only marginally larger than the eigenvalue spacing. The corresponding eigenvectors are fully delocalized. Similar results hold for the sum of two real symmetric matrices, conjugated by a Haar orthogonal matrix. This is a joint work with Laszlo Erdős and Kevin Schnelli.

Mario Kieburg (Bielefeld Univ.) - What is the relation between eigenvalues & singular values ?

The relation between eigenvalues and singular values of an arbitrary complex matrix is highly non-trivial. Only inequalities are known in the general case like Weyl's inequalities and Horn's inequalities. For random matrices the situation changes drastically in contrast to fixed matrices. It is well-known that there is no unique map from the joint density of the eigenvalues to the joint density of the singular values and vice versa. When assuming some additional conditions on the random matrix ensemble this statement has not to necessarily hold anymore. Indeed when assuming bi-unitary invariance of the random matrix ensemble we have very recently shown that there is a bijection between the joint densities of the eigenvalues and singular values. Additionally we could find an explicit representation of this map which drastically simplifies in the case of polynomial ensembles. I will report on this progress. Furthermore I will outline certain consequences on the relation of the spectral statistics which are encoded in the kernels of the corresponding determinantal point processes.

Gernot Akemann (Bielefeld Univ.) - Products of random matrices: dropping the independence

The subject of products of independent random matrices has seen considerable progress recently. This is due to the observation that its complex eigenvalues and its singular values form determinantal point processes that are integrable. In this talk I will drop the independence and present results for the singular values of the product of two linearly coupled random matrices as a first step. The resulting ensemble is no longer polynomial, but can be solved explicitly in terms of an integrable kernel of biorthogonal functions. The process interpolates between that of two independent and a single Gaussian random matrix. For genuine coupling in the local scaling regime at the origin we find back the Meijer G-kernel of Kuijlaars and Zhang which is universal. Sending the coupling to zero at a rate 1/N, where N is the matrix size, we find a one-parameter family of kernels that interpolates between the Bessel- and Meijer G-kernel for two independent matrices. In this limit the two matrices in the product become strongly coupled and almost adjoint to each other. This is joint work with Eugene Strahov.

Martin Zirnbauer (Köln Univ.) - Introduction to superbosonization

Karol Kozłowski (ENS Lyon) - Asymptotic expansion of the sinh-model with varying interactions

The sinh-model with varying interactions refers to the partition function

$$\int_{\mathbb{R}^N} \prod_{a<b}^{N} \Big\{ \sinh\big[N^{\alpha}\omega_1(\lambda_a-\lambda_b)\big] \sinh\big[N^{\alpha}\omega_2(\lambda_a-\lambda_b)\big] \Big\} \cdot \prod_{a=1}^{N} \Big\{ \mathrm{e}^{-N^{1+\alpha}V(\lambda_a)} \Big\} \cdot \mathrm{d}^N \lambda.$$

Such partition functions provide one with toy models of multiple integrals describing scalar products and certain correlation functions in quantum integrable models solvable by the quantum separation of variables. The application to the physics of these models demand to extract the large-N behaviour of such multiple integrals. Although such partition functions present certain structural similarities with β-ensembles, the determination of their large-N asymptotic expansions demands the introduction of several new ingredients. After discussing the context which gives rise to sinh-model partition functions in quantum integrable models, I will present the main features of the method that allows one to extract the large-N behaviour of the sinh-varying interactions partition functions. This method builds, on the one hand, on an analysis of Schwinger-Dyson equations and, on the other hand, on the Riemann--Hilbert problem approach to truncated Wiener--Hopf singular-integral equations. The results that I will present stem from a joint work with G. Borot and A. Guionnet.

Nikolai Kitanine (Dijon Univ.) - Form factor approach to the asymptotic analysis of correlation functions in massless quantum integrable models

We study the asymptotic behaviour of the equal-time and dynamical correlation functions of massless quantum integrable models. We develop an approach based on the summation of the appropriate (critical) form factors. We show that this approach gives rise to a number of new hypergeometric identities. These identities lead to the leading terms of the asymptotics for the two-point and multi-point correlation functions.

Vadim Gorin (MIT) - Largest eigenvalues in random matrix beta-ensembles: structures of the limit

Despite numerous articles devoted to its study, the universal scaling limit for the largest eigenvalues in general beta log-gases remains a mysterious object. I will present two new approaches to such edge scaling limits. The outcomes include a novel scaling limit for the differences between largest eigenvalues in submatrices and a Feynman-Kac type formula for the semigroup spanned by the Stochastic Airy Operator (based on joint work with M.Shkolnikov).

Sandrine Péché (Paris Diderot) - Universal versus Non-universal features in random matrix theory via deformed ensembles

We will review some recent results about universal/non universal asymptotic spectral properties of random matrices in the limit where the dimension grows to infinity. The talk focuses on deformed Gaussian ensembles of random matrices, which is the simplest ensemble of non Gaussian random matrix.

Yan Fyodorov (Queens Mary, London) - On the height and position of maxima of GUE characteristic polynomials

Log-Mod of the characteristic polynomial of GUE matrices is an example of a regularized log-correlated and asymptotically Gaussian processes. Exploiting the methods of statistical mechanics we aim to compute explicitly the distribution for the value and position of the global maximum of the modulus of GUE characteristic polynomial. To that goal we have employed a conjecture on the nature of the freezing phenomena seen in the statistical mechanics of log-correlated random energy models. Important role in our approach is played by conjectured high-temperature duality. We provide numerical evidence supporting our conjectures. The talk will be based on joint works with Nick Simm arXiv: 1503.07110 and with Pierre Le Doussal arXiv: 1511.04258.

Djalil Chafaï (Paris Dauphine) - At the edge of interacting particle systems

This talk presents some results and open questions about the edge of certain interacting particle systems, related to or inspired by random matrix models.

Alexander Moll (MIT) - Random Partitions and the Quantum Benjamin-Ono Hierarchy

Jack measures on partitions occur naturally in the study of continuum circular log-gases in generic background potentials V at arbitrary values \beta of Dyson's inverse temperature. Our main result is a law of large numbers (LLN) and central limit theorem (CLT) for Jack measures in the macroscopic scaling limit, which corresponds to the large N limit in the log-gas. Precisely, the emergent limit shape and macroscopic fluctuations of profiles of these random Young diagrams are the push-forwards along V of the uniform measure on the circle (LLN) and of the restriction to the circle of a Gaussian free field on the upper half-plane (CLT), respectively. At \beta=2, this recovers Okounkov's LLN for Schur measures (2003) and coincides with Breuer-Duits' CLT for biorthogonal ensembles (2013). Our limit theorems follow from an all-order expansion (AOE) of joint cumulants of linear statistics, which has the same form as the all-order 1/N refined topological expansion for the log-gas on the line due to Chekhov-Eynard (2006) and Borot-Guionnet (2012). To prove our AOE, we rely on the Lax operator for the quantum Benjamin-Ono hierarchy with periodic profile V exhibited in collective field variables by Nazarov-Sklyanin (2013). The characterization of the limit laws as push-forwards follows from factorization formulas for resolvents of Töplitz operators with symbol V due to Krein and Calderón-Spitzer-Widom (1958).

Alexander Soshnikov (UC Davis) - Spectral properties of products of large random matrices with independent entries

The first half of the talk will be devoted to joint results with Sean O'Rourke, David Renfrew, and Van Vu on spectral properties of products of elliptic random matrices. In the second half, I will discuss recent results by my Ph.D. student Phil Kopel on linear eigenvalue statistics in Ginibre ensembles and its generalizations.

Alexander Sodin (Tel Aviv Univ.) - Semi-classical analysis of non-self-adjoint Kac operators

We shall discuss non-self-adjoint Kac operators, and in particular the asymptotics of their largest eigenvalues in the semi-classical regime. Such operators appear in particular as transfer matrices of supersymmetric models which encode the spectral properties of one-dimensional random operators. Joint work with Margherita Disertori.

Posters – Abstracts

Tulasi Ram Reddy Annapareddy (IISC Bangalore) – On critical points of random polynomials

We choose two deterministic sequences of complex numbers, whose empirical measures converge to the same probability measure in complex plane. We make a sequence of polynomials whose zeros are chosen from either of sequences at random. We show that the limiting empirical measure of zeros and critical points agree for these polynomials. As a consequence, we show that when we randomly perturb the zeros of a deterministic sequence of polynomials, the limiting empirical measures of zeros and critical points agree. This result can be interpreted as an extension of earlier results where randomness is reduced. Pemantle and Rivin initiated the study of critical points of random polynomials. Kabluchko proved the result considering the zeros to be i.i.d. random variables.

Christophe Charlier (UC Louvain) – Thinned circular unitary ensembles and conditional probabilities

Consider the operation of thinning the CUE eigenvalues, which consists of re-moving each eigenvalue exp(i\theta_1) , ..., exp(i\theta_n) independently with probability $s \in (0, 1)$. Af- ter this process, there are some remaining eigenvalues exp(i\phi_1) , ..., exp(i\phi_m) , $m \leq n$, where m is now itself a random variable. A quantity of interest is the gap probability, namely the probability that all thinned eigenvalues lie in a arc γ of the unit cir- cle. Asymptotics for this probability have been computed as $n \to \infty$ for the two cases when s is a constant and when s decreases sufficiently fast to 0. We study the regime when s is going to 0 slower, and observe a transition between the two previous regimes. Joint work with Tom Claeys.

Tomazs Checinski (Bielefeld Univ.) – Spectral correlation functions of the sum of two independent complex Wishart matrices with unequal covariances

We study the sum of two statistically independent complex Wishart matrices which separately comprise non-trivial empirical covariance matrices. They model the simplest set of time series exhibiting time dependent spatial correlations. Thereby we also consider a certain limits where one of the empirical covariance matrices degenerate. This special case follows a determinantal point process. Our results are: a closed form for the k-point resolvent as a supermatrix integral and in the half degenerate case an explicit expression for the correlation kernel of the corresponding determinantal point process. Additionally, from the kernel the spectral density is calculated analytically and compared with Monte Carlo simulations.

Manuela Girotti (UC Louvain) – Smallest singular value distribution and large gap asymptotics for products of random matrices

We study the distribution of the smallest singular eigenvalues for the finite product of certain random matrix ensemble, in the limit where the size of the matrices becomes large. The limiting distributions that we will study can be expressed as Fredholm determinants of certain integral operators, and generalize in a natural way the extensively studied hard edge Bessel kernel determinant. We will express such quantities in terms of a 2x2 Riemann-Hilbert problem, and use this representation to obtain so-called large gap asymptotics.

Peter Nejjar (ENS Paris) – Competition interfaces and asymptotic independence at shocks

We study the asymptotic behavior of the so-called competition interface (I_n,J_n) in Last Passage Percolation (LPP). We show that at shocks, asymptotically (I_n-J_n) behaves like the difference of two independent random variables. This asymptotic independence is further applied to study the law of particle positions in TASEP at shocks : for random initial data, we rederive the product form of the limit law at the shock obtained by Péché, Ferrari and Corwin; for deterministic initial data, we extend earlier results to the multipoint case. Joint work with Patrik Ferrari.

Roman Riser (Holon) - Power spectrum analysis in quantum chaology and its relation to Töplitz determinants

We consider the power spectrum of the tuned CUE with an additional fixed charge at 1. It has been conjectured that in a chaotic quantum system the power spectrum behaves like $1/\omega$ whereas in a integrable system the dependency is $1/\omega^2$ where the frequency ω has to be much larger than 1 and much smaller than n, the number of particles. Since the chaotic system follows the statistic of random matrices, we use the tuned CUE model. Our calculation of the power spectrum gives the leading order of the large n asymptotics which turns out to be proportional to $n\log(n)$. Further we show how it is related to Toeplitz determinants and the solution of a Painlev´e VI equation.

Vedran Sohinger (ETH Zürich) – The Gross-Pitaevskii hierarchy on periodic domains

The Gross-Pitaevskii hierarchy is a system of infinitely many linear PDEs which occurs in the derivation of the nonlinear Schrodinger equation from the dynamics of many-body quantum systems. We will study this problem in the periodic setting. Even though the hierarchy is linear, it is non-closed, in the sense that the equation for the k^{th} density matrix in the system depends on the $(k+1)^{st}$ density matrix. This structure poses its challenges in the study of the problem, in particular in the understanding of uniqueness of solutions. Moreover, by randomizing in the collision operator, it is possible to use probabilistic techniques to study related hierarchies at low regularities. I will summarize some recent results obtained on these problems, partly in joint work with Philip Gressman, Sebastian Herr, and Gigliola Staffilani.

Yi Sun (MIT) – Laguerre and Jacobi analogues of the Warren process

We define Laguerre and Jacobi analogues of the Warren process. That is, we construct local dynamics on a triangular array of particles so that (1) the projections to each level recover the Laguerre and Jacobi eigenvalue processes of Konig-O'Connell and Doumerc and (2) the fixed time distributions recover the Laguerre and Jacobi corners processes. Our techniques extend and generalize the framework of intertwining diffusions developed by Pal-Shkolnikov. One consequence is a construction of a particle system with local interactions whose fixed time distributions recover the hard edge of random matrix theory.

Baris Ugurcan (Univ. Western Ontario) - Divisible Sandpile and the discrete biLaplacian Gaussian field

In the divisible sandpile model, an initial mass distribution composed of i.i.d. random variables on the vertices of an infinite vertex transitive graph is disseminated by a local toppling rule so as to make the mass at each vertex less or equal to 1. After establishing the "least action principle", "abelian property" and "conservation of density" for this model, we set out to show that an i.i.d divisible sandpile s, with $E[s] = 1$ and $0 < Var[s] < \infty$, on an infinite and vertex-transitive graph is almost surely not stabilizable. Observe that this result includes Z^d for all dimensions d as a particular case. Then, we give quantitative estimates for the divisible sandpile model on the discrete torus and relate the number of topplings to a discrete biLaplacian Gaussian field. This is joint work with Lionel Levine, Mathav Murugan and Yuval Peres.