Random matrices and related problems

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Description: In this lecture we will explore some aspects of random matrix theory. We will then see how the underlying mathematical structure can be applied to various models in mathematics and physics. There models span from directed percolation, random domino tiling, equilibrium and non-equilibrium statistical mechanics.

Schedule: Monday 10-12; MA 645.

References used in the preparation of the lecture notes:

- Lecture notes on the same framework [16, 6]
- My PhD thesis [2]
- The standard book on Random Matrices [8]
- Booklet on random matrices [7]
- Universality in Mathematics and Physics [1]
- Point processes [9]
- Determinantal point processes [14, 15, 6]
- Airy processes [10, 5] (papers) and my review [3]
- Tracy-Widom distribution and Painlevée II [17]
- Functional analysis [13, 11, 12]
- Application to the 3D-Ising corner [4]
- Application to the PNG droplet [10]

References

[1] P. Deift, *Universality for mathematical and physical systems*, arXiv:math-ph/0603038 (2006).

[2] P.L. Ferrari, *Polynuclear growth on a flat substrate and edge scaling of GOE eigenvalues*, Comm. Math. Phys. **252** (2004), 77–109.

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[5] K. Johansson, *Discrete polynuclear growth and determinantal processes*, Comm. Math. Phys. **242** (2003), 277–329.

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[12] M. Reed and B. Simon, *Methods of modern mathematical physics IV: Analysis of operators*, Academic Press, New York, 1978.

[13] B. Simon, *Trace ideals and their applications*, second edition ed., American Mathematical Society.

[14] A. Soshnikov, *Determinantal random point fields*, Russian Math. Surveys **55** (2000), 923–976.

[15] A. Soshnikov, *Determinantal random fields*, Encyclopedia of Mathematical Physics (J.-P. Francoise, G. Naber, and T. S. Tsun, eds.), Elsevier, Oxford, 2006, pp. 47–53.

[16] H. Spohn, *Exact solutions for KPZ-type growth processes, random matrices, and equilibrium shapes of crystals*, Physica A **369** (2006), 71–99.

[17] C.A. Tracy and H. Widom, *Level-spacing distributions and the Airy kernel*, Comm. Math. Phys. **159** (1994), 151–174.