Exam format

We will meet forty-five minutes before the exam. You will tell me one topic of your choice from the list below (call it "topic 1"). Then, I will give you two more topics from the list (topics 2 and 3).

You will then have forty-five minutes of preparation time. During this time, you may access any resources you like (a copy of the book Continuous Time Markov Processes by Liggett will be available to you). You may make written notes to bring with you into the exam. You will be examined on topic 1, and on either topic 2 or 3 (your choice). You will be expected to know the basic definitions related to these topics, and you will be expected to know how to prove the main results; I might ask you to sketch proofs, and I might also ask you to explain individual steps. I may also ask you to solve some problems.

The exam will take place at 60 Endenicher Allee, room 4.042. You may sit in room 3.041 during your preparation time.

Topics

1 Markov chains

1.1 The forward and backward equations

What are the Kolmogorov forward and backward equations, and what are they used for? What is the meaning of uniqueness for the KBE? What is the meaning of sub-stochasticity?

1.2 The probabilistic construction

What is the construction? How does it related to the KBE? What is the meaning of blow-up?

1.3 Long-term behavior

What are stationary and reversible measures? What are recurrence and transience? What are the relationships between these things?

2 Feller processes

2.1 From the process to the semigroup and the generator

Given a Feller process, how do you construct its semigroup? Given its semigroup, how do you construct its generator?

2.2 From the generator to the semigroup

Given the generator, how do you construct its semigroup?

2.3 Martingales, and constructing the process

Given a generator of its semigroup, how do you construct the corresponding Feller process? How are the generator and the process related to the existence of certain martingales?

2.4 Stationarity

What are stationary measures and ergodic measures? How can one check for stationarity using the generator? What happens when the state space is compact?

2.5 Diffusion processes and their generators

How can you check if a Feller process has continuous paths? How can we use the generator to construct "time-changed" processes? How can we use the generator to specify "boundary behavior"?

3 Spin systems

3.1 Existence and ergodicity

We proved sufficient conditions for existence and ergodicity of spin systems. What is it, and what are its applications?

3.2 Attractivity

What are attractive spin systems, and what does attractivity tell us about couplings? What does it tell us about stationary measures?

3.3 The voter model

What is the voter model? Is it ergodic? What are its stationary distributions? What duality relation does it satisfy?

3.4 The contact process

What is the voter model? Is it ergodic? What are its stationary distributions? What duality relation does it satisfy?