Institute for Applied Mathematics
Summer 2015
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"Stochastic Processes"

Exercise sheet 1

Hand in Tue 14.4.2015 during lecture break

Exercise 1 (Conditional probability) [*5 Pts.*]

Suppose you have five coins, two of which have heads on both sides, one tails on both sides and two are normal.

a) You close your eyes and randomly choose one coin and toss it. What is the probability that the bottom side is heads?

b) You open your eyes and see that the coin shows heads. What is the probability that the bottom side shows heads?

c) Now you close your eyes again and flip the same coin once more. What is the probability that the bottom side shows heads?

d) When you open your eyes again, you see that heads is up again. What is the probability that the bottom side shows heads?

e) Finally you put aside that coin and choose randomly one of the remaining coins and flip it. What is the probability that the face-up side is heads?

Exercise 2 (Conditional probability) [*5 Pts.*]

A while ago, the four brothers Joe, William, Jack and Averell moved out of their hometown to four different cities and barely see each other. Recently they managed that each two of them met without the others. Each meeting ended either in a quarrel with probability p or with eternal peace with probability $1 - p$. (The possible quarrel in one meeting is independent of the other meetings.) They are still interested in rumors about the next stagecoach ("Postkutsche"), but in their anger they only tell others the rumor if they haven't had a quarrel with them. Now if Joe hears a rumor, what is the probability that

a) Averell hears it?

b) Averell hears it, if Joe and William had a quarrel?

c) Averell hears it, if William and Jack had a quarrel?

d) Averell hears it, if he and Joe had a quarrel?

Exercise 3 (Normal approximation) *[5 Pts.]*

Estimate the error of a sum of rounded numbers in the following way. The numbers $R_1, \dots, R_n \in \mathbb{R}$ can be represented as $R_i = Z_i + E_i$, where $Z_i \in \mathbb{Z}$ is the rounded number and $E_i \in [-1/2, 1/2)$ the part that is lost. The difference between the real sum $\sum_{i=1}^n R_i$ and the rounded sum $\sum_{i_1}^n Z_i$ is the error $S_n := \sum_{i=1}^n E_i$. Assume the E_i are independend random variables which are uniformly distributed on $[-1/2, 1/2)$. Use the central limit theorem to get a bound $K > 0$ such that

$$\mathbb{P}(|S_{100}| < K) \approx 0.95.$$

Exercise 4 (Bivariate normal distribution) *[5 Pts.]*

Let the two random variables X and Y be jointly normal distributed with density

$$f(x, y) := \frac{1}{2\pi\sqrt{1-\varrho^2}} \exp\left(-\frac{1}{2(1-\varrho^2)}(x^2 - 2\varrho xy + y^2)\right).$$

Let $Z := \min\{X, Y\}$. Show that

$$\mathbb{E}(Z) = -\sqrt{\frac{1-\varrho}{\pi}}, \quad \mathbb{E}(Z^2) = 1.$$