Graduate Seminar Stochastic Analysis (WS 2018/19), Tuesdays 14 c.t., Room 1.007, Andreas Eberle
Mean-field models

Date	*Name*	*Topic*	*Literature*
		A. Basics on propagation of chaos	**Flandoli 5 and 6.1, Sznitman I, Méléard 2.1**
		A1. Propagation of chaos I	
		A2. Propagation of chaos II	
		B. First applications	**Sznitman II, Méléard 2.3 and 3**
		B1. Boltzmann equation	Méléard 2.3 and 3
		B2. Burgers equation and local time	Sznitman II
		C. Intermediate scale, spatial structure	**Flandoli 5 and 6.2**
		C1. Quasilinear PDE as scaling limit	Flandoli, Oelschläger (Z.W'theorie 1985)
		C2. Rapid stirring and reaction-diffusion equations	Durrett (Lecture Notes St. Flour, Ch. 8)
		D. Free energy, phase transitions, equilibration	
		D1. Invariant measures and phase transition	Dawson (J.Stat.Phys.1983), Herrmann/Tugaut (SPA2010)
		D2. Decay of free energy, gradient flow structure	Carrillo/McCann/Villani (Rev.Math.Iberoamer. 2003)
		D3. Coupling approach	Malrieu (Ann.Prob. 2003)
		E. Large deviations	**Dawson/Gärtner**
		E1. Large deviations I	DG, Reygner
		E2. Large deviations II	DG
		F. Particle methods in filtering and simulation	**Bain/Crisan Chapters 3, 9, 10, Del Moral/Doucet**
		F1. Particle filters in continuous time I	BC 3,9
		F2. Particle filters in continuous time II	BC 9
		F3. Partcile methods in discrete time	Del Moral/Doucet, BC 10
		G. Mean-field games	**Cardialiaguet, Carmona/Delarue**
		G1. Analytic approach of Lasry/Lions	Cardialiaguet
		G2. Probabilistic approach	Carmona/Delarue (Math.Fin.Econ., SIAM J.CO 2013)
		H. Neural networks and machine learning	
		H1. Hodgkin-Huxley and FitzHugh-Nagumo	Baladron et al (J.Math.Neuroscience 2012)
		H2. Applications in machine learning	

- **F. Flandoli: Elements in mathematical oncology,** http://users.dma.unipi.it/~flandoli/Padova_lectures_vers3.pdf
- **A.S. Sznitman: Topics in propagation of chaos, École d'été St.Flour 1989, Springer LNM 1464**
- **S. Méléard: Asymptotic behaviour of some interacting particle systems, in: Probab. models for nonlinear PDE, Springer LNM 1627**
- **Dawson/Gärtner: Large deviations, free energy fctl. and quasi-potential for a mean-field sytem of interact. diff., Memoirs AMS 398**
- **Bain/Crisan: Fundamentals of stochastic filtering, Springer 2007**
- **Del Moral/Doucet: Particle methods: An introduction with applications, https://hal.inria.fr/inria-00403917/document**
- **Cardialaguet: Notes on mean-field games (based on lectures of P.-L. Lions), https://www.ceremade.dauphine.fr/~cardaliaguet/**

In stochastic mean-field dynamics, there is a number N of components/particles which all interact with each other. The dynamics is described for example by a system of coupled stochastic differential equations. Under certain assumptions, a „propagation of chaos" property holds: If the particles are independent at time zero then in the limit as N tends to infinity, asymptotic independence holds at any fixed time t. In this case, the dynamics of a single particle can be asymptotically described by a stochastic differential equation where the coefficients depend on the law of the process, and, correspondingly, the asymptotic laws of the processes at time t satisfy a nonlinear partial differential equation.

In the seminar we will consider both classical and more recent results on propagation of chaos. We will also look at large deviations, free energy, and phase transitions for the limit equations. Moreover, we will consider several applications, in particular to numerical methods for filtering, to biological models, and to mean-field games.

Prerequisites: „Introduction to Stochastic Analysis"