S4F3 Graduate Seminar Applied Probability   Sommersemester 2019

Malliavin calculus and Monte Carlo methods on function spaces

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In this seminar we consider two different topics. Both have in common that they are based on analysis on a space of functions that is endowed with a probability measure.

The first topic is “Malliavin calculus”. Here one introduces analytic structures (gradient, divergence, Sobolev spaces) on the Wiener space. These are closely connected to variations of stochastic differential equations, i.e., differentiation of the solutions w.r.t. parameters. Malliavin calculus provides a probabilistic proof of Hörmander’s theorem on the existence of smooth densities for solutions to SDE, and it has applications to Monte Carlo methods in finance.

The final goal of the second part is to be able to define Markov Chain Monte Carlo methods directly on an (infinite dimensional) function space. To this end, we will first introduce Gaussian measures on Hilbert spaces, and Gaussian random fields. In Bayesian statistics, such Gaussian measures are frequently used as prior distributions, and the corresponding posteriors are measures that have a density w.r.t. the Gaussian prior. In the seminar we will see how to construct Markov processes in discrete and continuous time for which such probability measures are invariant. This has implications for the design of efficient MCMC methods in high dimensions.

References:

a. **Malliavin calculus**
- M. Hairer, *Advanced Stochastic Analysis,* Lecture Notes http://www.hairer.org/Teaching.html
- D. Nualart, *The Malliavin calculus and related topics*, 2nd ed., Springer (2005)

b. **Markov Chain Monte Carlo on function spaces**
- M. Dashti, A. Stuart, *The Bayesian approach to inverse problems,* http://arxiv.org/abs/1302.6989
- G. Da Prato: *An introduction to infinite-dimensional analysis,* Springer (2006)

Prerequisites: „Introduction to Stochastic Analysis“.